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DETERMINATION OF VELOCITY AND TEMPERATURE IN AN UNSTABLE SURFACE

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ABSTRACT

In this paper we formulate equations that are solvable numerically to determine the velocity and temperature in an unstable surface

General Terms

Unsteady Motion, Heat Generation, Thermal Radiation.

1. INTRODUCTION

The boundary layer theory had been presented in 1904 by prandlt (see Schlichting [1]) the problem of boundary layer flow and heat transfer over a moving surface is of interest in numerous industrial such as polymer extrusion processes where the object enters the fluid for cooling below a certain temperature, hot rolling, glass fiber, and paper production. Sakiadis [2] introduce the study of boundary layer flow over a continuous solid surface moving with constant velocity, the boundary layer flow caused by a stretching surface has drawn the attention of many researches. The dynamics of the boundary layer flow over a stretching surface originated from pioneering work of Crane [3] he examined the steady incompressible boundary layer flow of a Newtonian fluid caused by stretching of flat sheet which moves in its own plane with linear velocity due to the application of uniform stress. This problem is particularly interesting since an exact solution of the two dimensional Nervier-stokes equations has been obtained by crane [3]. Grubka and Bobba [4] extended this problem to include the energy equation. Gupta and Gupta [5] consider the case when the surface is permeable. The flow field of a stretching wall with a power low velocity variation was discussed by Banks [6]. Ali [7] and Elbashbeshy [8] extended the work of Banks [6] for a porous stretching surface with different values of the injection parameter. Elbashbeshy and Bazid [9-12] reanalyzed the stretching problem discussed earlier by Elbashbeshy [8] including variable viscosity, internal heat generation, suction /injection and porous medium. All of the above mentioned studies deal with stretching surface where the flows were assumed to be steady. Unsteady flows due to stretching surface has been considered by Devi *et al.* [13], Anderson et al. [14], Elbashbeshy and Bazid [15-16], Nazar *et al.* [17] and Ishak *et al.* [18].

Nanofluid is described as a fluid in which solid nanoparticles with the length scales of 1-100 nm are suspended in conventional heat transfer basic fluid. These nanoparticles enhance thermal conductivity and convective heat transfer coefficient of the base fluid significantly. Conventional heat transfer fluids such as oil, water and ethylene glycol mixture are poor heat transfer fluids because the thermal conductivity affects the heat transfer coefficient between the heat transfer medium and the heat transfer surface. Therefore numerous methods have been taken to improve the thermal conductivity of these fluids by suspending nano/micro sized particle materials in liquids. The term nanofluid refers to fluids in which nano-scale particles are suspended in the base fluid and it has been suggested by Choi [19]. There are many studies on the mechanism behind the enhanced heat transfer characteristics using nanofluids. The collection of papers on this topic is included in the book by Das *et. al.* [20] and in the review papers by Azizah *et al*. [21], Aminreza *et al.* [22], Nazar *et al.* [23] and Hamad [24], Oztop *et al*.[25] and Yacob *et al.* [26].

2. FORMULATION OF THE PROBLEM

Consider an unsteady, laminar, and incompressible nanofluid on a continuous moving surface. The fluid is a water based nanofluid containing three type of nanoparticles, either Cu (Copper) or Ag (Silver) or Al_2O_3 (Aluminum oxide). The nanoparticles are assumed to have a uniform shape and size. Moreover, it is assumed that both the fluid phase and nanoparticles are in thermal equilibrium state. As shown in fig. (1) the *x*-axis runs along the surface, and the *y*-axis is perpendicular to it.

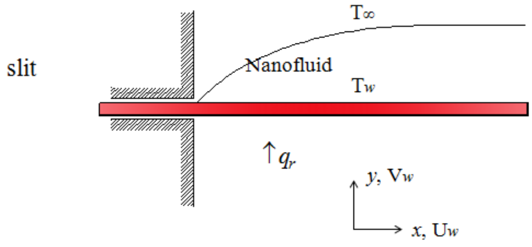

Fig (1). *physical model and coordinate system.*

The conservation equations for the unsteady boundary layer are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(\frac{\mu_{nf}}{\rho_{nf}}\right)\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}+Q\frac{T-T_\infty}{(\rho c_p)_{nf}}-\frac{1}{(\rho c_p)_{nf}}\frac{\partial q_r}{\partial y} \tag{3}$$

Subjected to the boundary conditions

$$\begin{aligned} &u=U_w,\quad v=v_w,\quad T=T_w, && at\, y=0\\ &u=0,\quad v=0,\quad T=T_\infty, && as\, y\to\infty \end{aligned} \tag{4}$$

Where u and v are velocity components in the x and y directions, respectively, t is the time, μ_{nf} is the nanofluid dynamic viscosity, ρ_{nf} is the density of the nanofluid, T is the temperature of the nanofluid, α_{nf} is the thermal diffusion of the nanofluid, Q is the heat source or sink, C_p specific heat of the nanofluid, T is the temperature of the nanofluid, T_w is the surface temperature, and T_∞ is ambient temperature, and q_r is the radiative heat flux.

The fluid is considered to be gray, absorbing-emitting radiation but non-scattering medium and the Rosselond approximation is used to describe the radiative heat flux in the energy equation (3). By using Rosseland approximation for radiation radiative heat flux is simplified as

$$q_r = -\frac{4\sigma}{3\alpha}\frac{\partial T^4}{\partial y} \qquad (5)$$

Where σ and α are the Stefen-Boltzman constant and the mean absorption coefficient respectively. assuming that the temperature differences within the flow are such that the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 in a Taylor series about T_∞ and neglecting higher order terms we get

$$T^4 \cong 4TT_\infty^3 - 3T_\infty^4 \qquad (6)$$

Using equation (5) and (6) in the energy equation (3) becomes

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma T_\infty^3}{3\alpha\left(\rho c_p\right)_{nf}}\frac{\partial^2 T}{\partial y^2} + \frac{Q}{\left(\rho c_p\right)_{nf}}\left(T - T_\infty\right) \qquad (7)$$

It is assumed that the velocity of the surface U_w and the surface temperature T_w are of the form

$$U_w(x,t) = \frac{ax}{(1-\gamma t)}, \qquad T_w(x,t) = T_\infty + \frac{bx}{(1-\gamma t)} \qquad (8)$$

Where a, b and γ are constants. The properties of nanofluid are defined as follows (see [16]).

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \quad \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \quad \alpha_{nf} = \frac{k_{nf}}{\left(\rho C_p\right)_{nf}}$$

$$\left(\rho C_p\right)_{nf} = (1-\phi)\left(\rho C_p\right)_f + \phi\left(\rho C_p\right)_s, \quad \frac{k_{nf}}{k_f} = \frac{\left(k_s + 2k_f\right) - 2\phi\left(k_f - k_s\right)}{\left(k_s + 2k_f\right) + \phi\left(k_f - k_s\right)}$$

where ϕ *is* nanoparticle volume fraction, it is worth mentioning that the study reduces to those of a viscous or regular fluid when (ϕ =0).

We look for similarity solution of eqs (1,2,and 7) subjected to the boundary conditions (4) of the following form

$$\eta = \sqrt{\frac{a}{\upsilon_f(1-\gamma t)}}\,y, \quad \psi = \sqrt{\frac{a\upsilon_f}{(1-\gamma t)}}\,x\,f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \qquad (9)$$

where η is the similarity variable and θ is the dimensionless temperature, υ_f is the kinamatic viscosity of the base (water), and ψ is the stream function, which defined as $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$ which satisfies equation (1), substituting

(9) into eqs. (2) and (7) we obtain

$$\left(\frac{1}{B}\right)f''' + ff'' - f'^2 - A\left(f' + \frac{\eta}{2}f''\right) = 0 \qquad (10)$$

$$\frac{\theta''}{p_r}\left[L + \frac{4R}{3\mathrm{M}}\right] + \left(\frac{\delta}{\mathrm{M}}\right)\theta + f\theta' - f'\theta - A\left(\theta + \frac{\eta}{2}\theta'\right) = 0 \qquad (11)$$

The boundary condition (4) become

$$f_{(0)} = f_w, \qquad f'_{(0)} = 1 \qquad \theta_{(0)} = 1$$

and

$$f'_{(\infty)} = 0, \qquad \theta_{(\infty)} = 0 \qquad (12)$$

where $B = (1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)$, $\mathrm{M} = \left(1-\phi+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)$ *and* $L = \frac{\left(\frac{k_{nf}}{k_f}\right)}{\left(1-\phi+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)}$, $A = \left(\frac{\gamma}{a}\right)$ is the unsteadiness parameter , $R = \frac{4\sigma T_\infty^3}{\alpha k_f}$ is the radiation parameter, $\delta = \frac{Qx}{U_w(\rho c_p)}$ is the heat source parameter and $p_r = \left(\frac{\upsilon\rho C_p}{k}\right)_f$ is the prandtl number, $fw = -V_w\sqrt{\frac{1-\gamma t}{a\upsilon_f}}$ is the suction/injection parameter

3. NUMERICAL SOLUTIONS AND RESULTS

We first convert the Equations (10) and (11) to a system of differential equations of first order, by using

$$S_1 = f, \; S_2 = f', \; S_3 = f'', \; S_4 = \theta, \; S_5 = \theta'$$

$$\left.\begin{aligned} S_1' &= S_2 \\ S_2' &= S_3 \\ S_3' &= B\left[S_2{}^2 - S_1S_3 + A\left(S_2 + \tfrac{1}{2}\eta S_3\right)\right] \\ S_4' &= S_5 \\ S_5' &= \left(\frac{3p_r\,\mathrm{M}}{4R+3\mathrm{M}\,L}\right)\left[S_2S_4 - S_1S_5 + A\left(S_4 + \tfrac{1}{2}\eta S_5\right) - \left(\frac{\delta}{\mathrm{M}}\right)S_4\right] \end{aligned}\right\} \qquad (13)$$

Subjected to the initial conditions

$$S_1(0) = fw, \;\; S_2(0) = 1, \;\; S_3(0) = m, \;\; S_4(0) = 1, \;\; S_5(0) = n \qquad (14)$$

where *m* and *n* are unknown to be determined as a part of the numerical solution.

Using mathematica, a function (F) has been defined such that F [*m*_, *n*_]:=NDSolve [system (13),(14)], The value of *m* and *n* are determined upon solving the equations, S2(ηmax) =0, and S4(ηmax) =0 to get the solution, NDSolve first searches for initial conditions that satisfy the equations, using a combination of Solve and a procedure much like Find Root. once *m* and *n* are determined the system (13) and (14) is closed, it can be solved numerically using the NDSolve function.

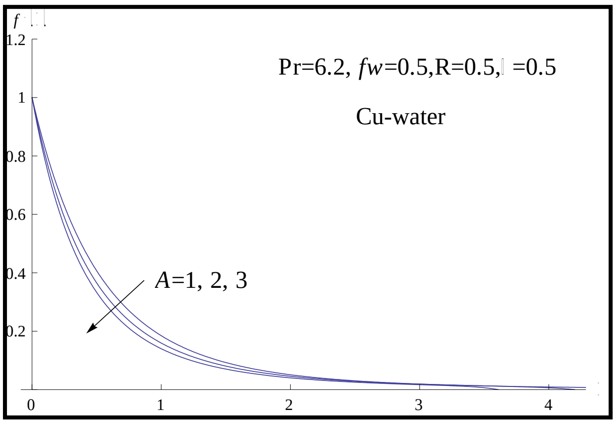

Fig (1). The Velocity profiles with increasing of unsteadiness parameter (A).

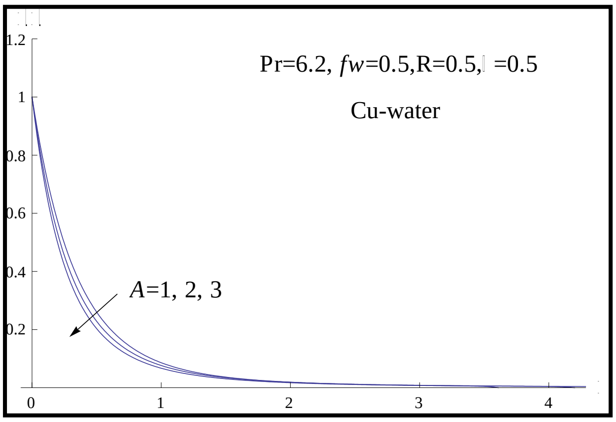

Fig (2). The Temperature profiles with increasing of unsteadiness parameter (A).

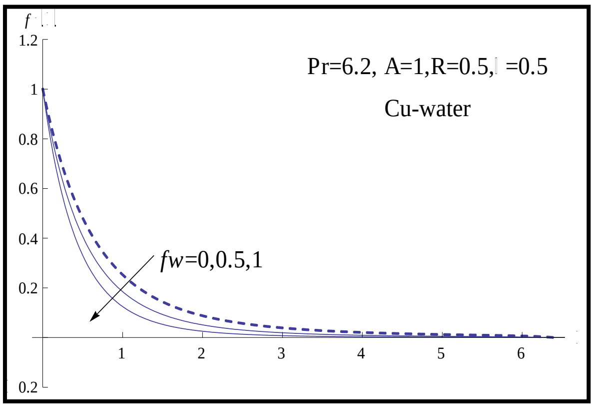

Fig (3). The Velocity profiles with increasing of suction parameter (fw).

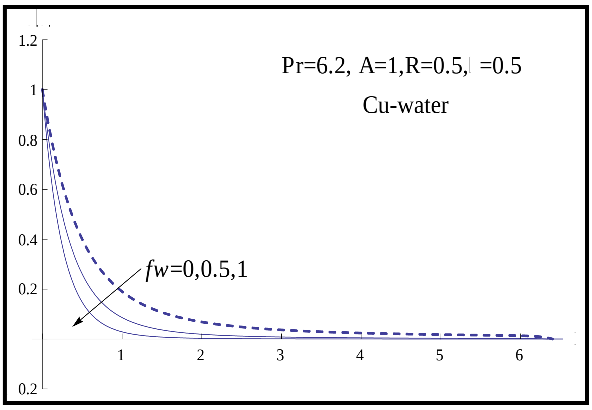

Fig (4). The Temperature profiles with increasing of suction parameter (fw).

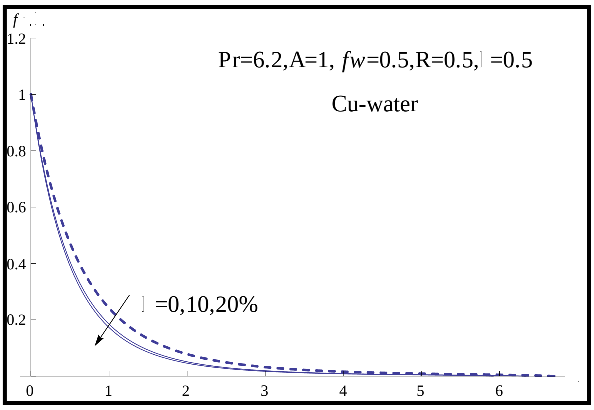

Fig (5). The Velocity profiles with increasing of nanoparticle volume fraction (ϕ).

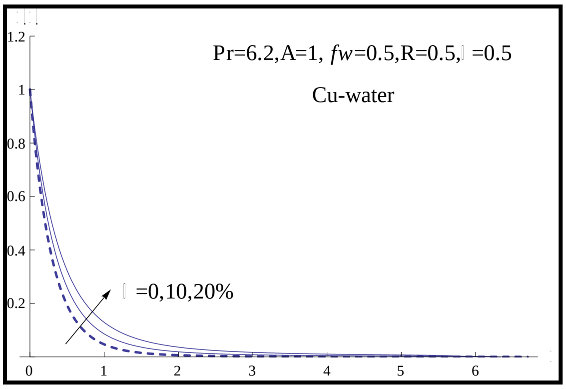

Fig (6). The Temperature profiles with increasing of nanoparticle volume fraction (ϕ).

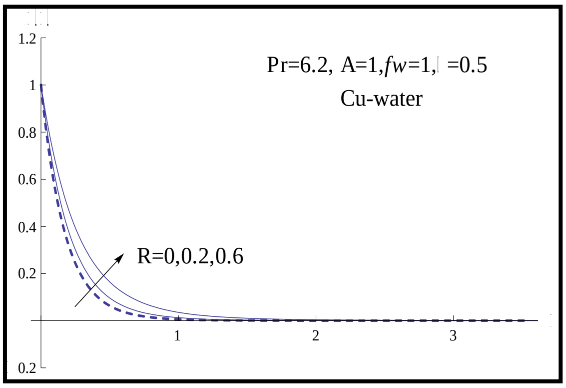

Fig (7). The Temperature profiles with increasing of radiation parameter (R).

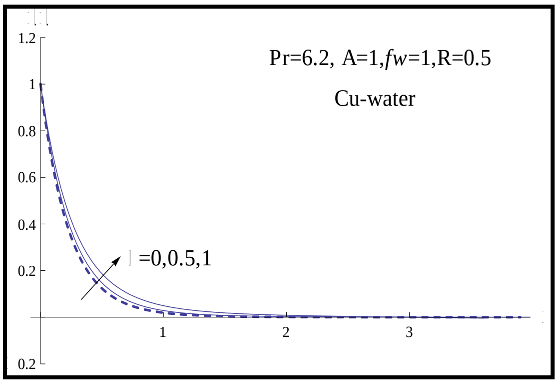

Fig (8). The Temperature profiles with increasing of heat source parameter (δ).

Table (1). Thermophysical properties of water and the elements Cu, Ag and Al2O3.

Properties	fluid (water)	Cu	Ag	Al2O3
Cp (j/kgK)	4179	385	235	765
ρ *(kg/m³)*	997.1	8933	10500	3970
K (W/mK)	0.613	400	429	40

To validate the numerical method used in this study, the case of (A =0,R= 0, $\delta = 0$ and ϕ =0) was considered in table (2) and the results for -θ'(0) are compared with the numerical solution which reported in Ishak *et a.l* [18]. Also the values of -*f* ''(0) at steady state are compared with the analytical solution which reported in Hamad [24] in table (3).

Table (2). *Values of - θ'(0) for a various values of (fw) and Pr at A=0, R =0 ,* $\delta = 0$, ϕ *=0.*

Pr	*Fw*	Ishak et al [18]	present results
1	-1.50	0.50000	0.50000
	-1.50	0.64520	0.64516
10	1.50	2.00000	2.00000
	1.50	16.08420	16.08422

Table (3). *Values of - f ''(0) for a various values of (* ϕ *) at Pr=6.2, A=0,R=0,* $\delta = 0$*, fw=0 for Cu- nanoparticles.*

ϕ	Hamad [24]	present results
0.05	1.10892	1.10892
0.1	1.17475	1.17474
0.15	120886	1.20888
0.2	1.21804	1.21804

From the engineering point of view, the most important characteristics of the flow are the skin friction coefficient, and Nusselt number which are indicate physically to surface shear stress, and rate of heat transfer respectively. This characteristics effect directly on the mechanical properties of the surface during heat treatment process, such that increasing the rate of heat transfer from the surface accelerates the cooling of the surface which improve the hardness and shear strength of the surface but on the other hand decrease the ductility of the surface and increase surface cracking.

a) surface shear stress

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\mu_f U_w}{(1-\phi)^{2.5}}\sqrt{\frac{a}{\upsilon_f(1-\gamma t)}}\, f''(0)$$

since the skin friction coefficient is given by

$$C_f = \frac{2\tau_w}{\rho U_w^2} \qquad \text{i.e} \qquad \frac{2f''(0)}{(1-\phi)^{2.5}} = C_f\sqrt{R_e}$$

b) surface heat flux

$$q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0} = -k_{nf}(T_w - T_\infty)\sqrt{\frac{a}{\upsilon_f(1-\gamma t)}}\ \theta'(0)$$

since the Nusselt number is given by

$$Nu = \frac{x q_w}{k_f(T_w - T_\infty)} \qquad \text{i.e} \qquad \frac{Nu}{\sqrt{R_e}} = -\frac{k_{nf}}{k_f}\theta'(0)$$

Table (3). values of velocity gradient and temperature gradient at the surface at ϕ =0.1, δ =0.5, Pr = 6.2.

A	fw	R	(Cu-Water) nano fluid				(Ag-Water) nano fluid				(Al2O3-Water) nano fluid			
			$-f''(0)$	$-\theta'(0)$	*Cfx*	*Nu*	$-f''(0)$	$-\theta'(0)$	*Cfx*	*Nu*	$-f''(0)$	$-\theta'(0)$	*Cfx*	*Nu*
1	-1	0	1.04967	1.13491	0.00386	1068.65	1.07770	1.11793	0.00397	1052.74	0.94324	1.14154	0.00347	1062.98
		0.6	1.04967	1.01620	0.00386	956.87	1.07770	0.99903	0.00397	940.78	0.94324	1.02367	0.00347	953.23
		1.2	1.04967	0.92873	0.00386	874.50	1.07770	0.91176	0.00397	858.60	0.94324	0.93658	0.00347	872.13
	1	0	2.34079	5.72400	0.00862	5389.77	2.48324	5.58073	0.00914	5255.31	1.87330	5.78023	0.00690	5382.46
		0.6	2.34079	3.81709	0.00862	3594.21	2.48324	3.71360	0.00914	3497.06	1.87330	3.86362	0.00690	3597.74
		1.2	2.34079	2.91084	0.00862	2740.88	2.48324	2.82628	0.00914	2661.48	1.87330	2.95685	0.00690	2753.37
2	-1	0	1.35502	1.69361	0.00499	1594.72	1.39471	1.67384	0.00513	1576.24	1.20673	1.70027	0.00444	1583.27
		0.6	1.35502	1.48878	0.00499	1401.85	1.39471	1.46938	0.00513	1383.70	1.20673	1.49483	0.00444	1391.96
		1.2	1.35502	1.34729	0.00499	1268.62	1.39471	1.32854	0.00513	1251.07	1.20673	1.35296	0.00444	1259.85
	1	0	2.61086	6.09234	0.00961	5736.61	2.76191	5.95168	0.01017	5604.64	2.11101	6.14051	0.00777	5717.95
		0.6	2.61086	4.16970	0.00961	3926.23	2.76191	4.07156	0.01017	3834.15	2.11101	4.20170	0.00777	3912.56
		1.2	2.61086	3.25502	0.00961	3064.96	2.76191	3.17741	0.01017	2992.13	2.11101	3.28247	0.00777	3056.58

Table (4). *values of velocity gradient and temperature gradient at the surface at fw=0.5,R=0.5,A=1, Pr = 6.1.*

ϕ	δ	(Cu-Water) nano fluid		(Ag-Water) nano fluid		(Al2O3-Water) nano fluid	
		$-\theta'(0)$	*Nu*	$-\theta'(0)$	*Nu*	$-\theta'(0)$	*Nu*
0	0	3.79357	2682.46	3.79357	2682.46	3.79357	2682.46
	0.5	3.41641	2415.77	3.41641	2415.77	3.41641	2415.77
	1	2.87500	2032.93	2.87500	2032.93	2.87500	2032.93
0.1	0	3.26473	3074.10	3.19896	3012.43	3.30056	3073.43
	0.5	2.88623	2717.70	2.80751	2643.80	2.93501	2733.04
	1	2.12295	1998.99	1.69357	1594.82	2.29154	2133.85
0.2	0	2.82234	3483.91	2.71197	3348.22	2.87304	3471.18
	0.5	2.45398	3029.19	2.31935	2863.48	2.52313	3048.43
	1	1.45842	1800.27	1.12963	1394.65	1.66750	2014.66

4. DISCUSSIONS

We present in this study a mathematical model of a moving continuous surface embedded into a nanofluid. The influence of thermal radiation (R), heat source (δ),suction/injection (*fw*), nanoparticles type, and nanoparticle volume fraction (ϕ),and the unsteadiness parameter (A) on the velocity and temperature within the boundary layer are shown in figures (1-8). We consider three different types of nanoparticles, Cu, Ag and Al_2O_3 with water as the base fluid. Table (1) shows the thermophysical properties of water and the elements Cu, Ag and Al_2O_3. The prandlt number of the base fluid (water) is kept constant at 6.2,

The effect of unsteadiness parameter (A) on the velocity and temperature within the boundary layer of (Cu-nanofluid) are shown in figures (1&2) It is observed that the velocity and temperature within the boundary layer decreases with increase of the unsteadiness parameter. also increase the unsteadiness parameter decrease the boundary layer thickness

Figures (3) and (4) show the effect of suction/injection parameter on the velocity and temperature within the boundary layer of Cu-nanofluid respectively. It is observed that the increases of suction/injection parameter decrease both the velocity and temperature within the boundary layer.

The effect of nanoparticles concentration (volume fraction) (ϕ) on the velocity and temperature within the boundary layer of Cu- nanofluid are shown in figures (5&6). It is observed that increases of nanoparticle volume fraction decrease the velocity but increase the temperature within the boundary layer.

The effect of thermal radiation parameter (R) and heat source parameter (δ) on the temperature within the boundary layer of (Cu-nanofluid) are shown in figures (7&8) respectively. It is observed that increase the thermal radiation and heat source increase the temperature within the boundary layer.

Table (3&4) show the values of velocity gradient and temperature gradient at the surface and the corresponding values of skin friction and Nusselt number for different values of (A), (*fw*), and (R) at $R_e = 5x10^5$. The effect of Nanoparticles type, nanoparticle concentration, steady and unsteady motion, suction and injection, and thermal radiation, heat generation on surface shear stress, surface heat flux and the mechanical properties (hardness, stiffness, strength, surface cracking, etc.) are discussed below.

A. type of nano particles

It is clear from table (3) that the values of velocity gradient at the surface increased gradually by changing the nanoparticle from Al_2O_3 to Cu to Ag. but the opposite effect occurs on temperature gradient. on the other hand the skin friction and surface shear stress are higher in the case of Ag-nanofluid than that in Cu and Al_2O_3-nanofluid, also the Nusselt number and rate of heat transfer from the surface are higher in the case of Cu-nanofluid than that in Al_2O_3 and Ag-nanofluid, which means that using Cu-nanofluid as a cooling medium is more useful for the surface hardness and strength.

B. concentration of nano particle within the base fluid

Table (4) shows that the values of velocity gradient at the surface increase gradually by increase the particle volume fraction from 10% to 20% in the case of Cu and Ag-nanoparticle and decrease for Al_2O_3 nanoparticle, But the temperature gradient decreases by increase of it for all types of Nanoparticles.

On the other hand the skin friction and Nusselt number both increase with increase of the concentration of nanoparticle within the base fluid. But in the presence of heat source the effect of nanoparticle concentration may be reversed such that increase the heat source parameter decrease the Nusselt number.

In general using a nanofluid in the cooling process is more active to improve the mechanical properties of the surface, such that using nanofluid increase the rate of heat transfer by (10-40%) more than in the case of pure water that leads to accelerate the cooling of the surface which increase the surface hardness and strength.

C. steady and unsteady motion

it is clear that the unsteady motion of the surface has a direct effect on the mechanical properties such that increase the unsteadiness parameter increase the velocity gradient, skin friction, and surface shear stress also increase the temperature gradient, Nusselt number, and rate of heat transfer. it is worth mentioning that increasing the unsteadiness parameter from 1 to 2 increases the Nusselt number by 6% in the case of suction and by 40% in the case of injection for all types of Nanoparticles.

D. suction and injection process

one can say that the suction/injection process play an important role in the cooling process, such that in the case of suction the velocity gradient, skin friction, surface shear stress, temperature gradient, Nusselt number and rate of heat transfer all are higher than that in the case of injection. As we know from previous the increase the rate of heat transfer from the surface improve the mechanical properties of surface.

E. Thermal radiation and heat generation

One can observe that increasing the thermal radiation and heat source both decrease the values of Nusselt number and rate of heat transfer that means the hardness and the strength of the surface will be decrease in the presence of both parameters .

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