Vol-1, Issue-2 PP. 232-242 ISSN: 2394-5788

GLOBAL JOURNAL OF ADVANCED RESEARCH
(Scholarly Peer Review Publishing System)

Exact Traveling Wave Solutions of Two Nonlinear Evolution Equation Via the $\exp(-\varphi(\xi))$-expansion Method

Jesmin Akter
Department of Applied Mathematics,
University of Rajshahi,
Bangladesh

M. Ali Akbar
Department of Applied Mathematics,
University of Rajshahi,
Bangladesh

ABSTRACT

The $\exp(-\varphi(\xi))$-expansion method is a promising method for finding exact traveling wave solutions to nonlinear evolution equations in physical sciences. In this article, we use the $\exp(-\varphi(\xi))$-expansion method to find the exact solutions for the nonlinear Zakharov-Kuznetsov-Benjamin-Bona-Mahony equation and the good Boussinesq equations. Many solitary wave solutions are formally derived. Being apparent, short and less limiting, this method can also be applied to many higher-dimensional NLEEs.

Keywords: $\exp(-\varphi(\xi))$-expansion method; Nonlinear evolution equation; Zakharov-Kuznetsov-Benjamin-Bona-Mahony equation; good Boussinesq equation; Homogeneous balance; Traveling wave solutions.

1. INTRODUCTION

Nonlinear evolution equations (NLEEs) hold great importance in several parts of mathematical and physical sciences. Obviously all the fundamental equations associated to physical and engineering problems are essentially nonlinear. A large amount of complex physical phenomena appears in fluid mechanics, quantum mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics can be described by NLEEs. In order to better understand the internal mechanism of nonlinear phenomena it is necessary to look for the exact traveling wave solution of NLEEs. Thus investigating exact traveling wave solutions of NLEEs is becoming successively attractive by the researchers who are concerned in nonlinear sciences. NLEEs are very difficult to solve evidently. In fact there are no general techniques that work for all such equations. Each individual equation has to be studied as a separate problem. For this reason, many new

GLOBAL JOURNAL OF ADVANCED RESEARCH
(Scholarly Peer Review Publishing System)

techniques for finding exact travelling wave solutions of NLEEs still have drawn a huge concentration by various groups of scientists. As a result, a lot of influential and significant methods have been established. Such as, the homogenous balance method [1, 2], the Hirota's bilinear transformation method [3, 4], the auxiliary equation method [5], the trial function method [6], the Jacobi elliptic function method [7], the tanh-function method [8-10], the homotopy perturbation method [11-13], the inverse scattering method [14], the sine-cosine method [15, 16], the truncated Painleve expansion method [17], the variational method [18-21], the Backlund transformation [22], the Exp-function method [23-25], the asymptotic method [26], the non-perturbative method [27], the (G'/G)-expansion method [28-35], the improved (G'/G) expansion method [36], the F-expansion method [37], the generalized Riccati equation [38] method, the Miura transformation [39], the extended F-expansion method [40], the weierstrass elliptic function method [41], the $\exp(-\varphi(\xi))$-expansion method [42, 43] and so on.

The objective of this article is, we will use the $\exp(-\varphi(\xi))$-expansion method to the nonlinear Zakharov-Kuznetsov-Benjamin-Bona-Mahony equation and the good Boussinesq equations. In the literature researched, these two equations have not been studied by this method. The solution procedure of this method is simple, explicit, and easily be extended to all kinds of NLEEs. The subject matter of this method is that the traveling wave solutions of a nonlinear evolution equation can be expressed by a polynomial in $\exp(-\varphi(\xi))$, where $\varphi(\xi)$ satisfies the ordinary differential equation (ODE):

$$\varphi'(\xi) = \exp(-\varphi(\xi)) + \mu \exp(\varphi(\xi)) + \lambda, \tag{1}$$

where $\xi = x - Vt$. The degree of the polynomial can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms and the coefficients of the polynomial can be obtained by solving a set of simultaneous algebraic equations.

Research on finding exact traveling wave solutions to the Zakharov-Kuznetsov-Benjamin-Bona-Mahony (ZKBBM) equation and good Boussinesq equations have been done by several researchers. As for example, Lanlan and Huaitang [44] engaged new (G'/G) expansion method to investigate the ZKBBM equation for constructing exact solutions, Mohyud-Din et al. [45] used the Exp-function method for obtaining solitary and periodic solutions of the good Boussinesq equation.

The consequence of this work is that the Zakharov-Kuznetsov-Benjamin-Bona-Mahony (ZKBBM) equation and the good Boussinesq equations are considered to construct new exact traveling wave solutions including solitons, periodic and rational solutions by applying the $\exp(-\varphi(\xi))$ expansion method.

The article is prepared as follows. In section 2, we describe briefly the $\exp(-\varphi(\xi))$-expansion method. In section 3, we apply this method to investigate the Zakharov-Kuznetsov-Benjamin-Bona-Mahony (ZKBBM) equation and the good Boussinesq equations. Finally in section 4, some important conclusions are given.

2. METHODOLOGY

In this section, we explain the $\exp(-\varphi(\xi))$-expansion method for finding traveling wave solutions of nonlinear evolution equations. Let us consider the nonlinear partial differential equation of the form

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, ...) = 0. \tag{2}$$

GLOBAL JOURNAL OF ADVANCED RESEARCH
(Scholarly Peer Review Publishing System)

Here $u(x,t)$ is an unknown function, P is a polynomial in $u(x,t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In order to solve (2) by using the $\exp(-\varphi(\xi))$-expansion method we have to complete the following steps:

Step1: Combining the real variables x and t by a compound variable ξ , we assume

$$u(x,t)=u(\xi),\xi=x-Vt, \tag{3}$$

where V is the speed of the travelling wave. Using the traveling wave variable (3), Eq. (2) changes to an ODE for $u=u(\xi)$:

$$Q(u,u',u'',u''',.....)=0, \tag{4}$$

where Q is a function of $u(\xi)$ and its derivatives, prime denotes the derivative with respect to ξ.

Step 2: Suppose the solution of (4) can be expressed by a polynomial in $\exp(-\varphi(\xi))$ as follows:

$$u(\xi)=\alpha_n(\exp(-\varphi(\xi)))^n+\alpha_{n-1}(\exp(-\varphi(\xi)))^{n-1}+\cdots \tag{5}$$

where $\alpha_n,\alpha_{n-1},\cdots$ and V are constants to be determined later such that $\alpha_n\neq 0$ and $\varphi(\xi)$ satisfies equation (1). The unwritten part of (5) is also a polynomial in $\exp(-\varphi(\xi))$.

Step 3: The positive integer n can be determined by considering the homogeneous balance between the highest order linear terms and nonlinear terms of the highest order appearing in (4). Our solutions now depend on the parameters involved in (1):

Case 1: $\lambda^2-4\mu>0$ and $\mu\neq 0$,

$$\varphi(\xi)=\ln\left\{\frac{1}{2\mu}\left(-\sqrt{\lambda^2-4\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+c_1)\right)-\lambda\right)\right\}. \tag{6}$$

where c_1 is a constant of integration.

Case 2: $\lambda^2-4\mu<0$ and $\mu\neq 0$,

$$\varphi(\xi)=\ln\left\{\frac{1}{2\mu}\left(-\lambda+\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+c_1)\right)\right)\right\}. \tag{7}$$

Case 3: $\mu=0$ and $\lambda\neq 0$,

$$\varphi(\xi)=-\ln\left\{\frac{\lambda}{\exp(\lambda(\xi+c_1))-1}\right\}. \tag{8}$$

Case 4: $\lambda^2-4\mu=0$, $\mu\neq 0$, and $\lambda\neq 0$,

GLOBAL JOURNAL OF ADVANCED RESEARCH
(Scholarly Peer Review Publishing System)

$$\varphi(\xi)=\ln\left\{-\frac{2(\lambda(\xi+c_1)+2)}{\lambda^2(\xi+c_1)}\right\}. \quad (9)$$

Case 5: $\mu=0$ and $\lambda=0$,

$$\varphi(\xi)=\ln(\xi+c_1). \quad (10)$$

Step 4: Inserting (5) into (4) and using (1), the left hand side is converted into a polynomial in $\exp(-\varphi(\xi))$. Equating each coefficient of this polynomial to zero, we obtain a set of algebraic equations for $\alpha_n,\cdots,V,\lambda$ and μ.

Step 5: Eventually solving the algebraic equations obtained in Step 4 with the aid of computer algebra system, we obtain the values of the constants $\alpha_n,\cdots,V,\lambda$ and μ. Substituting $\alpha_n,\cdots,V$ and the general solutions of (1) into solution (5), we obtain some valuable traveling wave solutions of (2).

3. APPLICATIONS OF THE METHOD

In this section, we utilize this method to obtain some new and more general exact travelling wave solutions of the ZKBBM equation and the good Bussinesq equations.

3.1: The ZKBBM Equation

Let us consider the ZKBBM equation in the form

$$u_t+u_x-2auu_x-bu_{xxt}=0. \quad (11)$$

Using the traveling wave transformation $\xi=x-Vt$, (11) is converted into the following ODE for $u=u(\xi)$:

$$-Vu'+u'-2auu'+bVu'''=0. \quad (12)$$

Eq. (12) is integrable, therefore integrating with respect to ξ, we obtain

$$C+(1\text{-}V)u-au^2+bVu''=0, \quad (13)$$

where the primes denote the derivatives with respect to ξ and C is an integration constant to be determined later. Considering the homogeneous balance between the highest-order derivative u'' and the nonlinear term u^2, we obtain $n=2$. Therefore, the solution of (13) is given by

$$u=\alpha_2(\exp(-\varphi(\xi)))^2+\alpha_1\exp(-\varphi(\xi))+\alpha_0, \quad (14)$$

where $\alpha_2\neq 0$. α_0 and α_1 are constants to be determined.

Using (1) from (14), we obtain

$$\begin{aligned}u''&=6\alpha_2(\exp(-\varphi(\xi)))^4+(2\alpha_1+10\alpha_2\lambda)(\exp(-\varphi(\xi)))^3\\&+(4\alpha_2\lambda^2+8\alpha_2\mu+3\alpha_1\lambda)(\exp(-\varphi(\xi)))^2\\&+(2\alpha_1\mu+6\alpha_2\mu\lambda+\alpha_1\lambda^2)(\exp(-\varphi(\xi)))+2\alpha_2\mu^2+\alpha_1\lambda\mu.\end{aligned} \quad (15)$$

GLOBAL JOURNAL OF ADVANCED RESEARCH
(Scholarly Peer Review Publishing System)

$$u^2 = \alpha_2^2\left(\exp(-\varphi(\xi))\right)^4 + 2\alpha_1\alpha_2\left(\exp(-\varphi(\xi))\right)^3 + \left(2\alpha_0\alpha_2 + \alpha_1^2\right)\left(\exp(-\varphi(\xi))\right)^2 + 2\alpha_1\alpha_0(-\varphi(\xi)) + \alpha_0^2. \quad (16)$$

Substituting (14)-(16) into (13) and collecting all terms with the same power of $\exp(-\varphi(\xi))$ together, the left hand is transformed into a polynomial in $\exp(-\varphi(\xi))$. Equating each coefficient of this polynomial to zero, we obtain an over-determined set of algebraic equations for $\alpha_1,\ \alpha_0,\ \lambda,\ \mu,\ C$ and V as follows:

$$-a\alpha_2^2 + 6b\,V\alpha_2 = 0.$$

$$10Vb\alpha_2\lambda + 2baV\alpha_1 - 2a\alpha_1\alpha_2 = 0.$$

$$\alpha_2 - V\alpha_2 + 4b\,V\alpha_2\lambda^2 + 3Vb\alpha_1\lambda - 2a\alpha_0\alpha_2 - a\alpha_1^2 + 8bV\alpha_2\mu = 0.$$

$$-V\alpha_1 - 2a\,\alpha_1\alpha_0 + 6Vb\alpha_2\lambda\mu + \alpha_1 + 2Vb\alpha_1\mu + bV\alpha_1\lambda^2 = 0.$$

$$C - a\alpha_0^2 - V\alpha_0 + \alpha_0 + 2bV\alpha_2\mu^2 + V\,b\,\alpha_1\lambda\mu = 0.$$

Solving the set of simultaneous algebraic equations by using the symbolic computation systems, such as Maple, we obtain the following solution:

$$C = \frac{b^2V^2\lambda^4 - 8b^2V^2\lambda^2\mu + 16b^2V^2\mu^2 - 1 + 2V - V^2}{4a},\ V = V,$$

$$\alpha_0 = \frac{bV\lambda^2 + 8bV\mu + 1 - V}{2a},\ \alpha_1 = \frac{6bV\lambda}{a},\ \alpha_2 = \frac{6bV}{a}, \quad (17)$$

where λ and μ are arbitrary constants.

By using (17) in (14), we obtain

$$u = \frac{6bV}{a}\exp(-\varphi(\xi))^2 + \frac{6bV\lambda}{a}\exp(-\varphi(\xi)) + \frac{bV\lambda^2 + 8bV\mu + 1 - V}{2a}, \quad (18)$$

where $\xi = x - Vt$.

Substituting the solutions of (1) in (18), we get four types of traveling wave solutions for the ZKBBM equation (11):

Type 1: When $\lambda^2 - 4\mu > 0$ and $\mu \neq 0$, we obtain the hyperbolic function traveling wave solution

$$u_1 = \frac{24bV\mu^2}{a}\left\{\lambda + \sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + c_1)\right)\right\}^{-2} - \frac{12bV\lambda\mu}{a}\left\{\lambda + \sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + c_1)\right)\right\}^{-1} + \frac{bV\lambda^2 + 8bV\mu + 1 - V}{2a},$$

where $\xi = x - Vt$.

Type 2: When $\lambda^2 - 4\mu < 0$ and $\mu \neq 0$, we obtain trigonometric solution

$$u_2 = \frac{24bV\mu^2}{a}\left\{\lambda - \sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi + c_1)\right)\right\}^{-2}$$

$$-\frac{12bV\lambda\mu}{a}\left\{\lambda - \sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi + c_1)\right)\right\}^{-1} + \frac{bV\lambda^2 + 8bV\mu + 1 - V}{2a}.$$

Type 3: When $\mu = 0$ and $\lambda \neq 0$, we obtain exponential solution

$$u_3 = \frac{6bV\lambda^2}{a}\frac{\exp\{\lambda(\xi + c_1)\}}{\{1 - \exp(\lambda(\xi + c_1))\}^2} + \frac{bV\lambda^2 + 1 - V}{2a}.$$

Type 4: When $\lambda^2 - 4\mu = 0$, $\lambda \neq 0$ and $\mu \neq 0$, we obtain rational function solution

$$u_4 = \frac{3bV\lambda^4(\xi + c_1)^2}{2a\{\lambda(\xi + c_1) + 2\}^2} - \frac{3bV\lambda^3(\xi + c_1)}{a\{\lambda(\xi + c_1) + 2\}} + \frac{bV\lambda^2 + 8bV\mu + 1 - V}{2a}.$$

Type 5: When $\mu = 0$ and $\lambda = 0$, we obtain rational function solution

$$u_5 = \frac{6bV}{a(\xi + c_1)^2} + \frac{1 - V}{2a}.$$

3.2: The Good Boussinesq Equation

Now we would like to construct traveling wave solutions to the good Boussinesq equation by the proposed method. Let us consider the good Boussinesq equation in the form

$$u_{tt} = -u_{xxxx} + u_{xx} + (u^2)_{xx} = 0. \tag{19}$$

The travelling wave variable $\xi = x - Vt$ permits us to change (19) into the following ODE:

$$(V^2 - 1)u'' + u'''' - (u^2)'' = 0, \tag{20}$$

where the primes indicate the derivatives with respect to ξ. Since Eq. (20) is integrable, therefore, integrating twice we obtain

$$C + (V^2 - 1)u + u'' - u^2 = 0, \tag{21}$$

where C is an integral constant that to be determined. Balancing the highest order linear term u'' and nonlinear term of the highest order u^2 in (21), we obtain $n = 2$.

Therefore, the solution of (21) is given by

$$u = \alpha_2(\exp(-\varphi(\xi)))^2 + \alpha_1\exp(-\varphi(\xi)) + \alpha_0, \tag{22}$$

GLOBAL JOURNAL OF ADVANCED RESEARCH
(Scholarly Peer Review Publishing System)

where $\alpha_2 \neq 0$. α_0, α_1, α_2 are constants to be determined.

From (22), by means of (1), we obtain

$$u'' = 6\alpha_2(\exp(-\varphi(\xi)))^4 + (2\alpha_1 + 10\alpha_2\lambda)(\exp(-\varphi(\xi)))^3 + (4\alpha_2\lambda^2 + 8\alpha_2\mu + 3\alpha_1\lambda)(\exp(-\varphi(\xi)))^2 + (2\alpha_1\mu + 6\alpha_2\mu\lambda + \alpha_1\lambda^2)(\exp(-\varphi(\xi))) + 2\alpha_2\mu^2 + \alpha_1\lambda\mu. \tag{23}$$

$$u^2 = \alpha_2^2(\exp(-\varphi(\xi)))^4 + 2\alpha_1\alpha_2(\exp(-\varphi(\xi)))^3 + (2\alpha_0\alpha_2 + \alpha_1^2)(\exp(-\varphi(\xi)))^2 + 2\alpha_1\alpha_0(-\varphi(\xi)) + \alpha_0^2. \tag{24}$$

Substituting (22)-(24) in (21) and collecting all terms of the same power of $\exp(-\varphi(\xi))$ together, the left hand side is converted into a polynomial in $\exp(-\varphi(\xi))$. Equating the coefficients of this polynomial to zero, yields a set of simultaneous algebraic equations for $\alpha_0, \alpha_1, \alpha_2, \lambda, \mu, C$ and V as follows:

$$6\alpha_2 - \alpha_2^2 = 0.$$
$$10\alpha_2\lambda + 2\alpha_1 - 2\alpha_1\alpha_2 = 0.$$
$$-\alpha_1^2 - 2\alpha_2\alpha_0 + V^2\alpha_2 + 8\alpha_2\mu + 3\alpha_1\lambda - \alpha_2 + 4\alpha_2\lambda^2 = 0.$$
$$6\alpha_2\lambda\mu - \alpha_1 + \alpha_1\lambda^2 - 2\alpha_1\alpha_0 + V^2\alpha_1 + 2\alpha_1\mu = 0.$$
$$C - \alpha_0 + V^2\alpha_0 + \alpha_1\lambda\mu - \alpha_0^2 + 2\alpha_2\mu^2 = 0.$$

Solving the set simultaneous algebraic equations, yields

$$C = -6\lambda^2\mu - \alpha_0^2 - 12\mu^2 + \alpha_0\lambda^2 + 8\mu\alpha_0,\ V = \pm\sqrt{-\lambda^2 + 1 - 8\mu + 2\alpha_0},$$
$$\alpha_0 = \alpha_0,\ \alpha_1 = 6\lambda,\ \alpha_2 = 6, \tag{25}$$

where λ and μ are arbitrary constants.

Substituting (25) into (22), we obtain

$$u(\xi) = 6(\exp(-\varphi(\xi)))^2 + 6\lambda\exp(-\varphi(\xi)) + \alpha_0, \tag{26}$$

where $\xi = x \mp \sqrt{-\lambda^2 + 1 - 8\mu + 2\alpha_0}\, t$.

Now making use of solutions (6)-(10) of (1) in (26), we obtain more traveling wave solutions of the good Boussinesq equation (19) as follows:

Type 1: When $\lambda^2 - 4\mu > 0$ and $\mu \neq 0$, we get hyperbolic function solution

GLOBAL JOURNAL OF ADVANCED RESEARCH
(Scholarly Peer Review Publishing System)

$$u_1 = 24\mu^2\left\{\lambda + \sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + c_1)\right)\right\}^{-2}$$
$$-12\lambda\mu\left\{\lambda + \sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + c_1)\right)\right\}^{-1} + \alpha_0\,,$$

where $\xi = x \mp \sqrt{-\lambda^2 + 1 - 8\mu + 2\alpha_0}\, t$, c_1 is an arbitrary constant.

Type 2: When $\lambda^2 - 4\mu < 0$ and $\mu \neq 0$, we obtain trigonometric solution

$$u_2 = 24\mu^2\left\{\lambda - \sqrt{4\mu - \lambda^2}\tan\left(\frac{\sqrt{4\mu - \lambda^2}}{2}(\xi + c_1)\right)\right\}^{-2}$$
$$-12\lambda\mu\left\{\lambda - \sqrt{4\mu - \lambda^2}\tan\left(\frac{\sqrt{4\mu - \lambda^2}}{2}(\xi + c_1)\right)\right\}^{-1} + \alpha_0$$

Type 3: When $\mu = 0$ and $\lambda \neq 0$, we obtain exponential solution

$$u_3 = \frac{6\lambda^2 \exp(\lambda(\xi + c_1))}{\{1 - \exp(\lambda(\xi + c_1))\}^2} + \alpha_0.$$

Type 4: When $\lambda^2 - 4\mu = 0$, $\lambda \neq 0$ and $\mu \neq 0$, we obtain rational function solution

$$u_4 = \frac{3\lambda^4(\xi + c_1)^2}{2\{\lambda(\xi + c_1) + 2\}^2} - \frac{3\lambda^3(\xi + c_1)}{\{\lambda(\xi + c_1) + 2\}} + \alpha_0.$$

Type 5: When $\mu = 0$ and $\lambda = 0$, we get rational function solution

$$u_5 = 6(\xi + c_1)^{-2} + \alpha_0\,,$$

where $\xi = x \mp \sqrt{-\lambda^2 + 1 - 8\mu + 2\alpha_0}\, t$, c_1 is an arbitrary constant.

4. CONCLUSIONS

In this article, we have successfully formulated solitary waves solutions using the traveling wave solutions for the ZKBBM equation and the good Boussinesq equation via the $\exp(-\varphi(\xi))$-expansion method. The wave solutions are obtained through the hyperbolic, trigonometric, exponential, and rational functions. All of our results have been verified with Maple, with respect to the original equation and found correct. The calculation procedure of this method is simple, direct and constructive. In particular we can say this method is quite efficient and much effective for finding exact solutions of NLEEs.

5. REFERENCES

[1] M. Wang, Solitary wave solutions for variant Boussinesq equations, Phys. Lett. A, 199 (1995) 169-172.

[2] E.M.E. Zayed, H.A. Zedan and K.A. Gepreel, On the solitary wave solutions for nonlinear Hirota-Satsuma coupled KdV equations, Chaos, Solitons Fract., 22 (2004) 285-303.

[3] R. Hirota, Exact envelope soliton solutions of a nonlinear wave equation, J. Math. Phys., 14 (1973) 805-810.

[4] R. Hirota and J. Satsuma, Soliton solution of a coupled KdV equation, Phys. Lett. A, 85 (1981) 407-408.

[5] Sirendaoreji, New exact travelling wave solutions for the Kawahara and modified Kawahara equations, Chaos Solitons Fract., 19 (2004) 147–150.

[6] M. Inc and D.J. Evans, On traveling wave solutions of some nonlinear evolution equations, Int. J. Comput. Math., 81 (2004) 191-202.

[7] A.T. Ali, New generalized Jacobi elliptic function rational expansion method, J. Comput. Appl. Math., 235 (2011) 4117-4127.

[8] M.A. Abdou, The extended tanh-method and its applications for solving nonlinear physical models, Appl. Math. Comput., 190 (2007) 988-996.

[9] E.G. Fan, Extended tanh-function method and its applications to nonlinear equations, Phys. Lett. A, 277 (2000) 212-218.

[10] W. Malfliet, Solitary wave solutions of nonlinear wave equations, Am. J. Phys., 60 (1992) 650- 654.

[11] S.T. Mohyud-Din, A. Yildirim and S. Sariaydin, Numerical soliton solutions of the improved Boussinesq equation, Int. J. Numer. Methods Heat Fluid Flow, 21(7) (2011) 822-827.

[12] S.T. Mohyud-Din, A. Yildirim and G. Demirli, Analytical solution of wave system in R^n with coupling controllers, Int. J. Numer. Methods Heat Fluid Flow, 21(2) (2011) 198-205.

[13] S.T. Mohyud-Din, A. Yildirim and S. Sariaydin, Numerical soliton solution of the Kaup–Kupershmidt equation, Int. J. Numer. Methods Heat Fluid Flow, 21(3) (2011) 272-281.

[14] M.J. Ablowitz and P.A. Clarkson, Solitons, Nonlinear Evolution Equations and Inverse Scattering Transform, Cambridge University Press, Cambridge, 1991,

[15] A.M. Wazwaz, A sine-cosine method for handing nonlinear wave equations. Math. Comput., 40 (2004) 499-508.

[16] S .Bibi, S.T.Mohyud-Din (2013) Traveling wave solutions of KdVs using sine-cosine method. J. Asso. Arab Univ. Basic and Appl. Sci., doi: org/10.1016/j.jaubas.2013.03.006 (in press).

[17] J.Weiss , M.Tabor and G.Carnevale , The Painleve property for partial differential equations, J. Math. Phys., 24 (1983) 522.

[18] J.H. He, Variational iteration method for delay differential equations. Commun Nonlinear Sci. Numer. Simulat., 2 (4) (1997) 235-236.

[19] M.A. Abdou and A.A. Soliman, New applications of variational iteration method. Phys. D, 211 (1-2) (2005) 1-8.

[20] S. Abbasbandy, Numerical solutions of nonlinear Klein-Gordon equation by variational iteration method. Internat. J. Numer. Meth. Engr., 70 (2007) 876-881.

[21] A.S. Arife and A. Yildirim, New modified variational iteration transform method (MVITM) for solving eighth-order boundary value problems in one step. World Appl. Sci. J., 13(10) (2011) 2186-2190.

[22] C. Rogers and W.F. Shadwick, Backlund Transformations, Academic Press, New York, 1982.

[23] J.H. He and X.H. Wu, Exp-function method for nonlinear wave equations, Chaos, Solitons and Fractals, 30 (2006) 700-708.

[24] M.A. Akbar, N.H.M. Ali, New Solitary and Periodic Solutions of Nonlinear Evolution Equation by Exp-function Method, World Appl. Sci. J., 17(12) (2012) 1603-1610.

[25] H. Naher, A.F. Abdullah and M.A.Akbar, New traveling wave solutions of the higher dimensional nonlinear partial differential equation by the Exp-function method, J. Appl. Math., Article ID 575387, (2012) 14 pages. doi: 10.1155/2012/575387.

[26] J.H. He, An elementary introduction to recently developed asymptotic methods and nano-mechanics in textile engineering, Int. J. Mod. Phys. B, 22(21) (2008) 3487-3578.

[27] J.H. He, Non-perturbative methods for strongly nonlinear problems, Dissertation. de-Verlag im Internet GmbH, Berlin, 2006.

[28] M. Wang, X. Li and J. Zhang, The (G'/G)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics, Phys. Lett. A, 372 (2008) 417-423.

[29] M.A.Akbar, N.H.M. Ali and E.M.E. Zayed, Abundant exact traveling wave solutions of the generalized Bretherton equation via (G'/G)-expansion method, Commun. Theor. Phys., 57 (2012) 173-178.

[30] M.A. Akbar, N.H.M. Ali and S.T. Mohyud-Din, Some new exact traveling wave solutions to the (3+1)-dimensional Kadomtsev-Petviashvili equation, World Appl. Sci. J., 16 (11) (2012) 1551-1558.

[31] E.M.E. Zayed, Traveling wave solutions for higher dimensional nonlinear evolution equations using the (G'/G)-expansion method, J. Appl. Math. Inform., 28 (2010) 383-395.

[32] E.M.E. Zayed and K.A. Gepreel, The (G'/G)-expansion method for finding the traveling wave solutions of nonlinear partial differential equations in mathematical physics, J. Math. Phys., 50 (2009) 013502-013514.

[33] M.A. Akbar and N.H.M. Ali, The alternative (G'/G)-expansion method and its applications to nonlinear partial differential equations, Int. J. Phys. Sci., 6 (35) (2011) 7910-7920.

[34] M.A. Akbar, N.H.M. Ali and S.T. Mohyud-Din, The alternative (G'/G)-expansion method with generalized Riccati equation: application to fifth order (1+1)-dimensional Caudrey-Dodd-Gibbon equation, Int. J. Phys. Sci., 7 (5) (2012) 743-752.

[35] A.R. Shehata, The traveling wave solutions of the perturbed nonlinear Schrodinger equation and the cubic-quintic Ginzburg Landau equation using the modified (G'/G)-expansion method, Appl. Math. Comput., 217 (2010) 1-10.

[36] J. Zhang, F. Jiang and X. Zhao, An improved (G'/G)-expansion method for solving nonlinear evolution equations, Int. J. Com. Math., 87(8) (2010) 1716-1725.

[37] M.L. Wang and X.Z. Li, Applications of F-expansion to periodic wave solutions for a new Hamiltonian amplitude equation, Chaos, Solitons and Fract., 24 (2005) 1257-1268.

[38] Z. Yan and H. Zhang, New explicit solitary wave solutions and periodic wave solutions for Whitham Broer-Kaup equation in shallow water, Phys. Lett. A, vol. 285 no.5-6 (2001) 355-362.

[39] T. L. Bock and M. D. Kruskal, A two-parameter Miura transformation of the Benjamin-One equation, Phys. Lett. A, 74 (1979) 173-176.

GLOBAL JOURNAL OF ADVANCED RESEARCH
(Scholarly Peer Review Publishing System)

[40] M. A. Abou, The Extended F-Expansion Method and Its Applications for a Class of Nonlinear Evolution Equation, Chaos, Solitons and Fract., 31 (2007) 95-104.

[41] N.A. Kudryashov, Exact Solutions of the Generalized Kuramoto-Sivashinsky Equation, Phys. Lett. A, 147(5-6) (1990) 287-291.

[42] K, Khan and M.A. Akbar, Application of $\exp(-\varphi(\xi))$-expansion Method to Find the Exact Solutions of Modified Benjamin-Bona-Mahony Equation, World Appl. Sci. J, 24(10) (2013) 1373-1377.

[43] J. Akter and M.A. Akbar, The $\exp(-\varphi(\xi))$-expansion method for exact solutions to the nonlinear KdV equation and the (2+1) dimensional Zakharov-Kuznetsov (ZK) equations, Elixir Appl. Math., **75** (2014) 27684-27692.

[44] X. Lanlan and C. Huaitang, New (G'/G)-Expansion Method and Its Applications to Nonlinear PDE, Phy. Review Research centre, 3(4) (2013) 407-415.

[45] S. T. Mohyud-Din, M.A. Noor and A. Waheed, "Exp-function method for generalized traveling solutions of good Boussinesq equations," J. Appl. Math. Comput., 30 (2009) 439-445.