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NEW OPERATIONS ON VAGUE IDEALS OVER LATTICES

K.Reena and Sr. I.Arockiarani
Department of Mathematics,
Nirmala College for women, Coimbatore,
India

ABSTRACT

In this paper we introduced some operations on Vague set of L and discussed some elementary results. Further, we applied these operations on Vague ideal of L and investigated their lattice structures.

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1. INTRODUCTION

In 1993 W.L.Gau and D.J.Buehrer[9] Proposed the theory of Vague sets as an improvement of theory of Fuzzy sets in approximating the real life situation. Vague sets are higher order Fuzzy sets. A Vague set A in the universe of discourse U is a Pair $(t_A, 1 - f_A)$ where t_A and f_A are Fuzzy subsets of U satisfying the Condition $t_A(u) \leq 1 - f_A(u)$ for all u ∈U. R.Biswas[7] initiated the study of Vague algebra by introducing the concepts of Vague groups, Vague normal groups. H.Khan , M.Ahmad and R.Biswas[12] introduced the notion of Vague relations and studied some properties of them. N.Ramakrishna[13,14] continued this study by studying Vague Cosets, Vague Products and several properties related to them. In 2008, Y.B.Jun and C.H.Park[11] introduced the notion of Vague Ideals in Substraction algebra. T.Eswarlal[8] had introduced the notion of Vague ideals and normal Vague ideals in Semirings in 2008. In 2005 K.Hur et.al[10] studied in detail the notion of intuitionistic Fuzzy Ideals of a ring and established their characterization in terms of level subsets. Moreover they studied the Lattice structure of intuitionistic Fuzzy Ideals of a ring and their Modularity. In this Paper we introduced the concept of Vague sublattices and Ideals in a Lattice. Their characterizations in terms of level subsets are provided and their homomorphic images under various conditions are studied.

2. PRELIMINARIES

Definition 2.1: [8]

A Vague set A in the universe of discourse S is a Pair (t_A, f_A) where t_A : S → [0,1] and f_A : S → [0,1] are mappings (called truth membership function and false membership function respectively) where t_A(x) is a lower bound of the grade of membership of x derived from the evidence for x and f_A(x) is a lower bound on the negation of x derived from the evidence against x and t_A(x) + f_A(x) ≤ 1 ∀x∈ S.

GLOBAL JOURNAL OF ADVANCED RESEARCH
(Scholarly Peer Review Publishing System)

Definition 2.2: [8]

The interval [t_A(x),1- f_A(x)] is called the Vague value of x in A, and it is denoted by V_A(x). That is V_A(x) = [t_A(x),1- f_A(x)].

Definition 2.3: [8]

A Vague set A of S is said to be contained in another Vague set B of S. That is A $\subseteq$ B, if and only if V_A(x) $\leq$ V_B(x). That is t_A(x)$\leq$ t_B(x) and $1 - f_A$(x)$\leq$ $1 - f_B$(x) $\forall$x $\in$ S.

Definition 2.4: [8]

Two Vague sets A and B of S are equal (i.e) A = B, if and only if A$\subseteq$B and B$\subseteq$A.

(i.e) V_A(x)$\leq$ V_B(x) and V_B(x)$\leq$ V_A(x) $\forall$x$\in$ S, which implies t_A(x) = t_B(x) and $1 - f_A$(x) = $1 - f_B$(x).

Definition 2.5 :[8]

The Union of two vague sets A and B of S with respective truth membership and false membership functions t_A, f_A and t_B, f_B is a Vague set C of S, written as C = A $\cup$ B, whose truth membership and false membership functions are related to those of A and B by t_C = max{t_A, t_B} and $1 - f_C$ = max {$1 - f_A$, $1 - f_B$ }=1-min{f_A, f_B}.

Definition 2.6: [8]

The Intersection of two vague sets A and B of S with respective truth membership and false membership functions t_A, f_A and t_B, f_B is a Vague set C of S, written as C = A $\cap$ B, whose truth membership and false membership functions are related to those of A and B by t_C = min{t_A, t_B} and $1 - f_C$ = min {$1 - f_A$, $1 - f_B$ }=1-max{f_A, f_B}.

Definition 2.7: [8]

A Vague set A of S with t_A(x) = 1 and f_A(x) = 0 $\forall$x$\in$ S, is called the unit vague set of S.

Definition 2.8: [8]

A Vague set A of S with t_A(x) = 0 and f_A(x) = 1 $\forall$x$\in$ S, is called the zero vague set of S.

Definition 2.9: [8]

Let A be a Vague set of the universe S with truth membership function t_A and false membership function f_A, for α,β $\in$ [0,1] with $\alpha\leq\beta$, the (α, β) cut or Vague cut of the Vague set A is a crisp subset $A_{(\alpha,\beta)}$ of S given by $A_{(\alpha,\beta)}$ = {x$\in$S: V_A(x)$\geq$ (α, β)},(i.e) $A_{(\alpha,\beta)}$ = {x$\in$S: t_A(x)$\geq\alpha$ and 1-f_A(x)$\geq$ β}

Definition 2.10: [8]

The α-cut, A_α of the Vague set A is the (α,α) cut of A and hence it is given by A_α = { x$\in$S : t_A(x) $\geq$ α}.

Definition 2.11: [10]

Let (X,$\leq$) be a Poset, if $\forall$ a,b$\in$S $\Rightarrow$ a$\vee$b, a$\wedge$b $\in$ X. Then (X,$\leq$) or (X,$\vee$,$\wedge$) is called a Lattice where a$\vee$b = $\vee${a,b}=sup{a,b} , a$\wedge$b = $\wedge${a,b} = inf{a,b}.

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Definition 2.12: [10]

Let (X,∨,∧) be a Lattice, if it satisfied following distributivity Laws, then it is called a distributive Lattice i) a∨(b∧c) = (a∨b)∧(a∨c) ,∀a,b,c ∈L ii) a∧(b∨c) = (a∧b)∨(a∧c) , ∀a,b,c ∈L

Definition 2.13: [10]

A Fuzzy subset μ of L is called a Fuzzy Sublattice of L if i) $\mu(x \vee y) \geq \min\{\mu(x), \mu(y)\}$

ii) $\mu(x \wedge y) \geq \min\{\mu(x), \mu(y)\}$ ∀ x,y ∈L

Definition 2.14: [10]

A Fuzzy subset μ of L is called a Fuzzy Sublattice of L if
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3. NEW OPERATIONS ON VAGUE IDEALS OVER LATTICES

Definition 3.1:

Let A,B ∈ VS(L). Then we define on VS(L) the following Operations.

i) A+B = {<z, V_{A+B}(z)>/z∈L}, Where $V_{A+B}(z) = \sup_{z=x\vee y}\{\min\{ V_A(x), V_B(y)\}\}$
ii) AB = {<z, V_{AB}(z)>/z∈L}, Where $V_{AB}(z) = \sup_{z=x\wedge y}\{\min\{ V_A(x), V_B(y)\}\}$
iii) A⊕B = {<z, $V_{A\oplus B}$(z)>/z∈L}, Where $V_{A\oplus B}(z) = \sup_{z\leq x\vee y}\{\min\{ V_A(x), V_B(y)\}\}$
iv) AoB = {<z, V_{AoB}(z)>/z∈L}, Where $V_{AoB}(z) = \sup_{z\geq x\vee y}\{\min\{ V_A(x), V_B(y)\}\}$
v) A•B = {<z, $V_{A\bullet B}$(z)>/z∈L}, Where $V_{A\bullet B}(z) = \sup_{z=\vee_{i=1}^{n} x_i\wedge y_i}\{\min\{ V_A(x_i), V_B(y_i)\}\}$

Lemma 3.2:

Let A, B, C ∈VS(L). Then the following conditions hold.

i) AB = BA, A+B = B+A, A•B = B•A
ii) AB⊆A•B⊆AoB
iii) C(A+B) ⊆CA+CB
iv) (C+B)A ⊆CA+BA
v) (A∩ B)C⊆AC∩BC
vi) A⊆B ⇒AC⊆BC and A•C⊆B•C
vii) A+B⊆A⊕B and AB⊆AoB, equality holds if L is distributive.
viii) A⊆A+A, A⊆AA, A⊆A⊕A, A⊆AoA and A⊆A•A.

Proof: Follows from definitions.

Lemma 3.3:

Let A,B∈VS(L) with $\sup_{x\in L} t_A(x) = t_1$, $\sup_{x\in L} t_B(x) = t_2$ and $\sup_{x\in L} 1 - f_A(x) = k_1$, $\sup_{x\in L} 1 - f_B(x) = k_2$. Then A⊆A⊕B ⇒ $t_1 \le t_2$, $k_1 \le k_2$.

Proof:

Suppose $t_1 > t_2$ and $k_1 > k_2$. Then $\sup_{x\in L} t_B(x) < \sup_{x\in L} t_A(x) \Rightarrow \sup_{x\in L} t_B(x) < \sup_{x\in L} t_A(z_0)$ for some z_0∈L. So that $t_{A\oplus B}(z_0) = \sup_{z_0\le x\vee y}\{\min\{t_A(x), t_B(y)\} \le \sup_{z_0\le x\vee y} t_B(y) \le \sup_{y\in L} t_B(y) < t_A(z_0)$. This contradicts A⊆ A⊕B. Therefore $t_1 \le t_2$. And $\sup_{x\in L}(1 - f_B(x)) < \sup_{x\in L}(1 - f_A(x)) \Rightarrow \sup_{x\in L}(1 - f_B(x)) < \sup_{x\in L}(1 - f_A(w_0))$ for some w_0∈L. Then $(1 - f_{A\oplus B}(w_0)) = \sup_{w_0\le x\vee y}\{\min\{1 - f_A(x)), 1 - f_B(y)\} \le \sup_{w\le x\vee y}(1 - f_B(y)) \le \sup_{y\in L}(1 - f_B(y)) < (1 - f_A(w_0))$. This contradicts A⊆ A⊕B. Therefore $k_1 \le k_2$.

Lemma 3.4:

Let A,B∈VS(L) with $\sup_{x\in L} t_A(x) = t_1$, $\sup_{x\in L} t_B(x) = t_2$ and $\sup_{x\in L} 1 - f_A(x) = k_1$, $\sup_{x\in L} 1 - f_B(x) = k_2$. Then A⊆A⊕B and B⊆A⊕B ⇒ $t_1 = t_2$, $k_1 = k_2$.

Proof: Follows from 3.3

Lemma 3.5:

Let A,B∈VS(L) with $\sup_{x\in L} t_A(x) = \sup_{x\in L} t_B(x) = t$ and $\sup_{x\in L} 1 - f_A(x) = \sup_{x\in L} 1 - f_B(x) = k$. Then

i) A,B ⊆A⊕B , if A and B both attain their Sup for t and Sup for 1-f.
ii) A,B ⊆A⊕B , if A and B both do not attain their Sup for t and Sup for 1-f.

Proof:

i) Suppose that A and B both attain their Sup for t and Sup for 1-f. Let $\sup_{x\in L} t_A(x) = t_A(x_0)$ and $\sup_{x\in L} t_B(x) = t_B(y_0)$ for some x_0, y_0∈L and $\sup_{x\in L} 1 - f_A(x) = 1 - f_A(l_0)$ and $\sup_{x\in L} 1 - f_B(x) = 1 - f_B(m_0)$ for some l_0, m_0∈L . Then by our assumption $t_A(x_0) = t_B(y_0)$ and $1 - f_A(l_0) = 1 - f_B(m_0)$. For z∈L, we have $t_{A\oplus B}(z) = \sup_{z\le x\vee y}\{\min\{t_A(x), t_B(y) \ge \min\{t_A(z), t_B(y_0)\}$ as $z \le z\vee y_0 = t_A(z)$, since $t_A(z), \le \sup_{x\in L} t_A(x) = t_A(x_0) = t_B(y_0)$ and $1 - f_{A\oplus B}(z) = \sup_{z\le x\vee y}\{\min\{1 - f_A(x), 1 - f_B(y) \ge \min\{1 - f_A(z), 1 - f_B(m_0)\}$ as $z \le z\vee m_0 = 1 - f_A(z)$, since $1 - f_A(z) \le \sup_{x\in L} 1 - f_A(x) = 1 - f_B(m_0)$. Hence A⊆A⊕B. Similarly we can prove that B⊆A⊕B .

ii) Suppose that A and B both do not attain their Sup for t and Sup for 1-f. Since A do not attain its Sup for t and 1-f, we have $t_A(z) < t$ ∀z∈L and $1 - f_A(z) < k$ ∀z∈L. Then there exist y_0∈L Such that $t_B(y_0) > t_A(z)$ and there exist l_0∈L such that $1 - f_B(l_0) > 1 - f_A(z)$ But $z \le z\vee y_0$ and hence $t_{A\oplus B}(z) = \sup_{z\le x\vee y}\{\min\{t_A(x), t_B(y) \ge \min\{t_A(z), t_B(y_0)\} = t_A(z)$ and $z \le z\vee l_0$, we have $1 - f_{A\oplus B}(z) = \sup_{z\le x\vee y}\{\min\{1 - f_A(x), 1 - f_B(y) \ge \min\{1 - f_A(z), 1 - f_B(l_0)\} = 1 - f_A(z)$. So that A⊆A⊕B. Similarly, we can prove that B⊆A⊕B.

Proposition 3.6:

Let A ∈VS(L). Then A is a VL of L if and only if A+A =A and AA=A.

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Proof:

We have A⊆A+A and A⊆AA. Let A is a VL of L. Then ∀x,y∈L such that z=x∨y, we have $V_A(z)$ = V_A(x∨y)≥min{ $V_A(x)$, $V_A(y)$}. Therefore $V_A(z) \geq \underset{z=x\vee y}{sup}$ {min{$V_A(x)$, $V_A(y)$}} = $V_{A\oplus A}(z)$. Hence A⊇A+A. Thus A=A+A. Now , ∀x, y∈L such that z=x∧y, we have $V_A(z)$ = V_A(x∧y)≥min{ $V_A(x)$, $V_A(y)$}. Therefore $V_A(z) \geq$ $\underset{z=x\wedge y}{sup}$ {min{$V_A(x)$, $V_A(y)$}} = $V_{AA}(z)$. Thus A⊇AA. Hence A=AA. Conversely, suppose that A=A+A and A=AA. Then ∀x,y∈L. we have V_A(x∨y) = V_{A+A}(x∨y)= $\underset{x\vee y=x_1\vee y_1}{sup}${min{$V_A(x_1)$, $V_A(y_1)$}}≥ min{$V_A(x)$,$V_A(y)$} and V_A(x∧y) = V_{AA}(x∧y)= $\underset{x\wedge y=x_1\wedge y_1}{sup}${min{$V_A(x_1)$, $V_A(y_1)$}}≥ min{$V_A(x)$,$V_A(y)$}. Hence A is a VL of L.

Proposition 3.7:

Let A ∈VS(L). Then A ∈VI(L) if and only if A⊕A = A.

Proof:

Suppose A ∈VI(L). Let z∈L, choose x,y∈L such that z≤x∨y. Then $V_A(z) \geq V_A$(x∨y)≥min{$V_A(x)$, $V_A(y)$}, since A VI of L. So that $V_A(z) \geq \underset{z\leq x\vee y}{sup}$ {min{$V_A(x)$, $V_A(y)$}} = $V_{A\oplus A}(z)$. Hence A⊇A⊕A. Clearly A⊆A⊕A. Thus A = A⊕A. Conversely suppose that A = A⊕A. Let x,y∈L. Then V_A(x∨y) = $V_{A\oplus A}$(x∨y)= $\underset{x\vee y=x_1\vee y_1}{sup}${min{$V_A(x_1)$, $V_A(y_1)$}} ≥ min{$V_A(x)$,$V_A(y)$} and V_A(x∧y) = $V_{A\oplus A}$(x∧y)= $\underset{x\wedge y=x_1\wedge y_1}{sup}${min{$V_A(x_1)$, $V_A(y_1)$}} ≥ min{$V_A(x)$,$V_A(y)$}. Hence A is a VL of L. Now let z_1, z_2∈ L such that $z_1 \leq z_2$. Then $V_A(z_2)$ = $V_{A\oplus A}(z_2)$ = $\underset{z_2\leq x_2\vee y_2}{sup}${min{$V_A(x_2)$, $V_A(y_2)$}}, x_2, y_2∈L ≤ $\underset{z_1\leq x_1\vee y_1}{sup}${min{$V_A(x_1)$, $V_A(y_1)$}} , as $z_1 \leq z_2$ = $V_A(z_1)$. Thus $V_A(z_2) \leq V_A(z_1)$. Hence A is a VI of L.

Theorem 3.8:

Let A, B∈VI(L). Then A⊕B∈VI(L).

Proof:

Suppose that for some x, y∈L $t_{A\oplus B}$(x∨y) <min{ $t_{A\oplus B}$(x), $t_{A\oplus B}$(y)}. Let $t_{A\oplus B}$(x∨y) = m_0. Then $m_0 < t_{A\oplus B}$(x) and $m_0 < t_{A\oplus B}$(y). This implies there exist a,b ∈ L such that x≤a∨b, m_0< min{$t_A(a)$, t_B(b)} and there exist c,d∈L such that y≤c∨d, m_0< min{$t_A(c)$, t_B(d)}. So that m_0< $t_A(a)$, m_0< t_B(b), m_0< $t_A(c)$ and $m_0 < t_B$(d). Hence $m_0 <$ min{$t_A(a)$, $t_A(c)$} ≤ $t_A(a$∨c), since A a VI of L. Also m_0 < min{t_B(b), t_B(d)}≤ t_B(b∨d), since B a VI of L. Thus $t_{A\oplus B}$(x∨y) ≥ $\underset{x\vee y\leq p\vee q}{sup}$ {min{$t_A(p)$, $t_B(q)$}}, p,q∈L. ≥ min{$t_A(a\vee c)$, t_B(b∨d)} > t_0. This contradicts $t_{A\oplus B}$(x∨y) = t_0. Hence $t_{A\oplus B}$(x∨y) ≥ min{ $t_{A\oplus B}$(x), $t_{A\oplus B}$(y)}, ∀x,y∈L...........(1). Similarly we can prove that $1 - f_{A\oplus B}$(x∨y) ≥ min{1- $f_{A\oplus B}$(x),1- $f_{A\oplus B}$(y)}, ∀x,y∈L.............(2). Again suppose that $t_{A\oplus B}$(x∧y)<max{$t_{A\oplus B}$(x), $t_{A\oplus B}$(y)}, for some x,y∈L. Let k_0 = $t_{A\oplus B}$(x∧y) and max{$t_{A\oplus B}$(x), $t_{A\oplus B}$(y)} = $t_{A\oplus B}$(x),(say). Then $k_0 < t_{A\oplus B}$(x). This implies there exist a,b∈L such that x≤a∨b and min{$t_A(a$), t_B(b) }>k_0. So that $t_{A\oplus B}$(x∧y) = $\underset{x\wedge y\leq p\wedge q}{sup}$ {min{$t_A(p)$, $t_B(q)$}}, p,q∈L ≥ min {$t_A(a)$, t_B(b)} >k_0 . This contradicts k_0 = $t_{A\oplus B}$(x∧y). Consequently $t_{A\oplus B}$(x∧y)≥max{$t_{A\oplus B}$(x), $t_{A\oplus B}$(y)}, for some x,y∈L.........(3). Similarly we can prove 1- $f_{A\oplus B}$(x∧y)≥max{1-$f_{A\oplus B}$(x), $1 - f_{A\oplus B}$(y)}, for some x,y∈L.---------------(4). Thus from (1),(2),(3),(4) A⊕B∈VI(L).

Theorem 3.9:

Let A,B∈VI(L) with $\underset{x\in L}{sup}$ V_A(x) = $\underset{x\in L}{sup}$ V_B(x) and both A, B attain the sup of t and 1-f.[or both A,B do not attain the sup of t and 1-f]. Then A⊕B is a VI generated by A and B.

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Proof:

By Lemma 3.5 and Theorem 3.10, A⊕B is a VI containing both A and B. Let C∈VI(L) such that A⊆C and B⊆C. Then for z∈L, we have $V_{A\oplus B}$(z) = $\sup_{z\leq x\vee y}$ {min{$V_A(x)$, $V_B(y)$}}≤ $\sup_{z\leq x\vee y}$ {min{$V_C(x)$, $V_C(y)$}} = $t_{C\oplus C}$(z) = $t_C(z)$, by Proposition 3.9 Hence A⊕B⊆C. Thus A⊕B is the least VI containing A and B. We denote the set of all VI's of L that attain both sup(t) = m and sup(1-f) = k by $VI_{[m,k]}(L)$ and the set of all VI's of L that do not attain both the sup(t) = m and sup(1-f) = k by $VI_{(m,k)}(L)$.

Theorem 3.10:

The set $VI_{(m,k)}(L)$ [$VI_{[m,k]}(L)$] forms a Lattice under the ordering ⊆ with join and meet defined by A∨B = A⊕B and A∧B = A∩B.

Proof:

We know that A∨B = A⊕B. Now we show that A⊕B ∈ $VI_{(m,k)}(L)$. Let z∈L. Then $t_{A\oplus B}$(z) = $\sup_{z\leq x\vee y}$ {min{$t_A(x)$, $t_B(y)$}}≤ min{$\sup_{x\in L}$ t_A(x) , $\sup_{x\in L}$ t_B(y) } = l. We show that 'l' is the least upper bound of $t_{A\oplus B}$. Let ε>0, since l is the supremum of t_A and t_B, there exist x_1, y_1 ∈L such that $t_A(x_1)$ >t-ε and $t_B(y_1)$>t-ε. So that for z_1 = $x_1\vee y_1$, we have $t_{A\oplus B}(z_1)$ = $\sup_{z_1\leq x\vee y}$ {min{$t_A(x)$, $t_B(y)$}}≥min{$t_A(x_1)$, $t_B(y_1)$ }>l-ε. Hence l is the least upper bound of $t_{A\oplus B}$. Similarly, we can prove m is the least upper bound of 1-$f_{A\oplus B}$. Thus A⊕B ∈$VI_{(m,k)}(L)$. Clearly , A∩B ∈$VI_{(m,k)}(L)$. Thus ($VI_{(m,k)}(L)$, ⊆, ⊕, ∩) forms a Lattice.

Theorem 3.11:

If L is distributive then the lattice $VI_{(m,k)}(L)$ [$VI_{[m,k]}(L)$] is distributive.

Proof:

Let A,B,C ∈ $VI_{(m,k)}(L)$. Then by distributive inequality, which is satisfied by every Lattice, we have A∧(B∨C)⊇(A∧B)∨(A∧C). The proof of the theorem will be complete if we show that A∧(B∨C)⊆(A∧B)∨(A∧C). If possible, suppose A∧(B∨C) ⊄(A∧B)∨(A∧C) then there exist z∈L such that $V_{A\wedge(B\vee C)}(z)$> $V_{(A\wedge B)\vee(A\wedge C)}(z)$. Since L is distributive A⊕B = A+B, so A∨B = A+B. So that $V_{A\cap(B+C)}(z)$> $V_{(A\cap B)+(A\cap C)}(z)$. Now, $V_{A\cap(B+C)}(z)$> $V_{(A\cap B)+(A\cap C)}(z)$ implies min{$V_A(z)$, $V_{B+C}(z)$} >$\sup_{z=x\vee y}$ {min{$V_{A\cap B}(x)$, $V_{A\cap C}(y)$}}. This implies min{$V_A(z)$, $\sup_{z=x\vee y}$ {min{$V_B(x)$, $V_C(y)$}} >$\sup_{z=x\vee y}$ {min{$V_{A\cap B}$ (x), $V_{A\cap C}(y)$}}. Then there exist x_0 ,y_0 ∈L such that z = $x_0\vee y_0$ and $V_A(z)$, min{$V_B(x_0)$, $V_C(y_0)$ } > $\sup_{z=x\vee y}$ {min{$V_{A\cap B}(x)$, $V_{A\cap C}(y)$}} ≥ min{$V_{A\cap B}(x_0)$, $V_{A\cap C}(y_0)$ } = min{$V_A(x_0)$, $V_B(x_0)$, $V_A(y_0)$, $V_C(y_0)$}. Hence min {$V_A(x_0)$, $V_B(x_0)$, $V_A(y_0)$, $V_C(y_0)$} = $V_A(x_0)$ or $V_A(y_0)$. So that $V_A(z)$> $V_A(x_0)$ or $V_A(y_0)$. But since A ∈$VI_{(m,k)}(L)$ and x_0, y_0≤z we have $V_A(x_0)$≥ $V_A(z)$ and $V_A(y_0)$≥ $V_A(z)$. Thus, $V_A(z)$> $V_A(x_0)$ or $V_A(y_0)$ is not true. So our assumption is wrong, consequently A∧(B∨C)⊆(A∧B)∨(A∧C). Hence $VI_{(m,k)}(L)$ is a distributive Lattice.

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