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LOCAL FUNCTIONS ON π-OPEN SETS IN IDEAL TOPOLOGICAL SPACES

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ABSTRACT

The focus of this paper is to define the local function on π-open set called π-local function and to introduce a new class of set operator $()^{*}$utilizing π-open neighbourhood namely the set operator $()^{*\pi}$ in ideal topological spaces. We derive several characterizations and properties of such function and operator in detail.

Keywords: π-open set, π-closed set, π-local function, set operator Ψ^{π}, ideal topological space

1. INTRODUCTION

The concept explored via ideals has a lengthy and interesting historic development. Kuratowski [10] introduced the concept of ideal topological spaces. The notion of Kuratowski operator plays a vital role in defining ideal topological space which has its application in localization theory in set topology by Vaidyanathaswamy [13]. Ideals have been frequently used in the fields closely related to topology such as real analysis measure theory and lattice theory. In 1990, Jankovic and Hamlett [6, 7] developed new topologies from old via ideals and introduced I-open sets with respect to an ideal I in 1992. In this paper, analogously to the local function $A^{*}(\mathcal{I}, \tau)$, we define π-local function with respect to $\mathcal{I}$ and τ. Moreover we introduce a set operator Ψ^{π} in ideal topological spaces and study their properties.

2. PRELIMINARIES

Throughout this paper (X, τ) is a topological space on which no separation axioms are assumed unless explicitly stated. The notation $(X, \tau, \mathcal{I})$ will denote the topological space (X, τ) and an ideal $\mathcal{I}$ on X with no separation properties assumed. For $A \subseteq (X, \tau)$, Cl(A) and Int(A) respectively denote the closure and interior of A with respect to τ. $N(x)$ denotes the open neighbourhood system at a point $x \in X$ and P(X) denotes the power set of X.

Definition: 2.1[10]

An ideal $\mathcal{I}$ on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies the following properties: (1) $A \in \mathcal{I}$ and $B \subseteq A$ implies $B \in \mathcal{I}$.

(2) $A \in \mathcal{I}$ and $B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$.

An ideal topological space is a topological space (X, τ) with an ideal $\mathcal{I}$ on X and is denoted by $(X, \tau, \mathcal{I})$.

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Definition: 2.2[10]

For a subset A of X, $A^*(\mathcal{I}) = \{x \in X: U \cap A \notin \mathcal{I}$ for each neighbourhood U of $x\}$ is called the local function of A with respect to $\mathcal{I}$ and τ. We simply write A^* instead of $A^*(\mathcal{I})$.

Definition: 2.2[10]

It is well known that $Cl^*(A) = A \cup A^*$ defines a Kuratowski closure operator for $\tau^*(\mathcal{I})$ which finar than τ.

Definition: 2.3[10]

A basis $\beta(\mathcal{I}, \tau)$ for $\tau^*(\mathcal{I})$ can be described as follows: $\beta(\mathcal{I}, \tau) = \{U - E: U \in \tau$ and $E \in \mathcal{I}\}$.

Definition: 2.4

A subset A of an ideal topological space (X, τ, I) is

(1) ∗-perfect [5], if $A = A^*$

(2) ∗- closed [6], if $A^* \subseteq A$

(3) *-dense [8], if $Cl^*(A) = X$

(4) τ^*-closed set [6], if $A = Cl^*(A)$

Definition: 2.5[15]

A subset A of a space (X, τ) is said to be regular open set, if A = int(cl(A)).

Definition: 2.6[13]

Finite union of regular open sets in (X, τ) is π-open in (X, τ).The complement of π-open in (X, τ) is π-closed in (X, τ).

Definition: 2.7[9]

Let (X, τ, $\mathcal{I}$) be an ideal topological space and A be a subset of X. Then $A_*(\mathcal{I}, \tau) = \{x \in X \mid A \cap U \notin \mathcal{I}$ for every $U \in SO(X, x)\}$ is called the semi local function of A with respect to $\mathcal{I}$ and τ, where $SO(X, x) = \{U \in SO(X) \mid x \in U\}$.

Definition: 2.8[1]

Let (X, τ, $\mathcal{I}$) be an ideal topological space. For a subset A of X, we define the following operator:
$\Gamma(A)(\mathcal{I}, \tau) = \{x \in X \mid A \cap cl(U) \notin \mathcal{I}$ for every $U \in \tau(x)\}$ is called the local closure function of A with respect to $\mathcal{I}$ and τ, where $\tau(x) = \{U \in \tau: x \in U\}$.

Definition: 2.9[2]

Given a space (X, τ, $\mathcal{I}$), a set operator $()^{*p}$: P(X) → P(X) is called the pre-local function of $\mathcal{I}$ with respect to τ is defined as follows; for $A \subseteq X$, $(A)^{*p}(\mathcal{I}, \tau) = \{x \in X / U_x \cap A \notin \mathcal{I}, for\ every$
$U_x \in PN(x)\}$, where $PN(x)\} = \{U \in PO(x) \mid x \in U\}$.

Lemma: 2.10[6]

Let (X, τ, I) be an ideal space and A, B subsets of X.

(1). If $A \subset B$, then $A^* \subset B^*$.

(2). If $G \in \tau$, then $G \cap A^* \subset (G \cap A)^*$

(3). $A^* = Cl(A^*) \subset Cl(A)$

3. 3. π-LOCAL FUNCTIONS

Definition: 3.1

Given a space (X, τ,$\mathcal{I}$), a set operator $()^{*\pi}$: P(X) → P(X) is called the π-local function of $\mathcal{I}$ with respect to τ is defined as follows: for $A \subseteq X$, $(A)^{*\pi}(\mathcal{I},\tau) = \{x \in X / U_x \cap A \notin \mathcal{I}, for\ every\ U_x \in \pi N(x)\}$, where $\pi N(x)\} = \{U \in \pi O(x) \mid x \in U\}$.We write π-local function as $A^{*\pi}(\mathcal{I})$ or $A^{*\pi}$ or $A^{*\pi}(\mathcal{I}, \tau)$.

Preposition: 3.2

Every local function is π-local function.

Proof:

Every π-open set is open.

Therefore $U \in \pi N(x) \Longrightarrow U \in N(x)$.

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Let $x \in A^* \Rightarrow U_x \cap A \notin \mathcal{I}, for\ every\ U_x \in N(x)\}$
$\Rightarrow U_x \cap A \notin \mathcal{I}, for\ every\ U_x \in \pi N(x)\}$
$\Rightarrow x \in A^{*\pi}$
$\Rightarrow A^* \subseteq A^{*\pi}$

Hence every local function is π-local function.

Remark: 3.3

π-local function need not be local function as shown in the following example.

Example: 3.4

X = {a, b, c,d}, τ = {X, ϕ, {a}, {b}, {a,b}, {a, b, d}} and I = {ϕ, {a}}.Take A = {c}. Then A^* = {c} and $A^{*\pi}$ = {c,d}. Thus {c,d} ⊈ {c}. Hence $A^{*\pi} \nsubseteq A^*$.

Remark: 3.5

The collection of all π-open subsets of a space (X, τ) fails to form an ideal on X.

Example: 3.6

X = {a, b, c, d}, τ = {X, ϕ, {d}, {a,b}, {a,b,d}} and I = {ϕ, {a}, {b}, {a,b}}.

Here {a, b} is π-open set, but {a} and {b} are not π-open sets. Therefore the collection of all π-open subsets does not form an ideal.

Remarks: 3.7

(1) For an ideal topological $(X, \tau, \mathcal{I})$, $\tau^{*\pi}(\mathcal{I})$ be the topology on X generated by the basis $\{U - E: U \in \pi O(x) \text{ and } E \in \mathcal{I}\}$.

(2) The closure operator in $\tau^{*\pi}(\mathcal{I})$ denoted by $Cl^{*\pi}$ is defined as follows: for $A \subseteq X$,
$Cl^{*\pi}(A) = A \cup A^{*\pi}$.

Lemma: 3.8

$\tau^{*\pi}(\mathcal{I}) = \{U \subseteq X: Cl^{*\pi}(X - U) = X - U\}$

Proof:

Let $\mathcal{A} = \{U - E: U \in \pi O(x) \text{ and } E \in \mathcal{I}\}$

$Cl^{*\pi}(X - U) = X - U \Leftrightarrow (X - U)^{*\pi} \subseteq X - U$
$\Leftrightarrow U \subseteq X - (X - U)^{*\pi}$

Therefore $x \in U \Rightarrow x \notin (X - U)^{*\pi}$. Then there exists $V \in \pi N(x)$, $V \cap (X - U) \in \mathcal{I}$. Let $\mathcal{I} = V \cap (X - U)$ then $x \in V - \mathcal{I} \subseteq U$ where $V \in \pi N(x)$ and $\mathcal{I} \in \mathcal{I}$. But $\tau^{*\pi}$ is the topology with $\mathcal{A}$ as a basis. Therefore $\tau^{*\pi}(\mathcal{I}) = \{U \subseteq X: Cl^{*\pi}(X - U) = X - U\}$.

Corollary: 3.9

For $A \subseteq (X\ \tau, \mathcal{I})$ we have

(1) If $\mathcal{I} = \{\phi\}$ then $A^{*\pi}(\{\phi\}) = \pi\text{-}Cl(A)$ and $Cl^{*\pi}(A) = \pi\text{-}Cl(A)$

(2) If $\mathcal{I} = P(X)$ then $A^{*\pi}(P(X)) = \phi$ and $Cl^{*\pi}(A) = A$. Hence in this case $\tau^{*\pi}(\mathcal{I})$ is the discrete topology.

Remark: 3.10

$\tau^{*\pi}(\mathcal{I}) \subseteq \tau^*(\mathcal{I})$. But the converse need not be true.

Example: 3.11

Consider the topology τ = {X, ϕ, {a}, {b}, {a, b}, {a, b, c}} on a set X = {a, b, c, d} and $\mathcal{I}$ = {ϕ, {a}}. Therefore $\tau^*(\mathcal{I})$ = {X, ϕ, {a}, {b}, {a, b}, {b, c}, {a, b, c}, {b, c, d}} and $\tau^{*\pi}(\mathcal{I})$ = {X, ϕ, {a}, {b}, {a, b}, {b, c, d}}. Hence $\tau^*(\mathcal{I}) \nsubseteq \tau^{*\pi}(\mathcal{I})$.

Theorem: 3.12

Let $(X, \tau, \mathcal{I})$ be a space and A, B ⊆ X then the following statements hold.

(1) $\phi^{*\pi}(\mathcal{I}) = \phi$

(2) If $A \subseteq B$ then $A^{*\pi}(\mathcal{I}) \subseteq B^{*\pi}(\mathcal{I})$

(3) $A^{*\pi}(\mathcal{I}) = \pi\text{-}Cl(A^{*\pi}(\mathcal{I})) \subseteq \pi\text{-}Cl(A)$

(4) $(A \cup B)^{*\pi}(\mathcal{I}) = A^{*\pi}(\mathcal{I}) \cup B^{*\pi}(\mathcal{I})$

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(5) $(A\cap B)^{*\pi}(\mathcal{I}) \subseteq A^{*\pi}(\mathcal{I}) \cap B^{*\pi}(\mathcal{I})$
(6) $A^{*\pi}(\mathcal{I}) - B^{*}(\mathcal{I}) \subseteq (A - B)^{*\pi}(\mathcal{I})$
(7) $(A^{*\pi})^{*\pi}(\mathcal{I}) \subseteq A^{*\pi}(\mathcal{I})$
(8) If $E \in \mathcal{I}$ then $(A\cup E)^{*\pi}(\mathcal{I}) = A^{*\pi}(\mathcal{I}) = (A-E)^{*\pi}(\mathcal{I})$
(9) If $U \in \pi O(X)$ then $U\cap A^{*\pi}(\mathcal{I}) = U\cap (U\cap A)^{*\pi} \subseteq (U\cap A)^{*\pi}(\mathcal{I})$

Proof:

(1) $\phi^{*\pi}(\mathcal{I}) = \{x\in X/ U_x \cap \phi \notin \mathcal{I}, for\ every\ U_x \in \pi N(x)\} = \phi$.Therefore $\phi^{*\pi}(\mathcal{I}) = \phi$.

(2) Let $A \subseteq B$.
$A^{*\pi}(\mathcal{I}) = \{x\in X/ U_x \cap A \notin \mathcal{I}, for\ every\ U_x \in \pi N(x)\}$.
$x \in A^{*\pi}(\mathcal{I}) \Rightarrow U_x \cap A \notin \mathcal{I}, for\ every\ U_x \in \pi N(x)$
$\Rightarrow U_x \cap B \notin \mathcal{I}, for\ every\ U_x \in \pi N(x)$
This shows that $x \in B^{*\pi}(\mathcal{I})$. Hence $A^{*\pi}(\mathcal{I}) \subseteq B^{*\pi}(\mathcal{I})$.

(3) To prove $A^{*\pi}(\mathcal{I}) = \pi\text{-}Cl(A^{*\pi}(\mathcal{I})) \subseteq \pi\text{-}Cl(A)$
$\pi\text{-}Cl(A^{*\pi}(\mathcal{I}))$ is the smallest π-closed set containing $A^{*\pi}$. Therefore $A^{*\pi} \subseteq \pi\text{-}Cl(A^{*\pi})$.
To prove $A^{*\pi} \supseteq \pi\text{-}Cl(A^{*\pi})$. Let $x \in \pi\text{-}Cl(A^{*\pi})$. Therefore every $U_x \in \pi N(x)$ intersects $A^{*\pi}$.

Thus $U_x \cap A^{*\pi} \neq \phi$. Let $y \in U_x \cap A^{*\pi}$. Then $U_x \in \pi N(y)$ and $y \in A^{*\pi} \Rightarrow U_x \cap A^{*\pi} \notin \mathcal{I}$ which implies that $x \in A^{*\pi}$. Therefore $\pi\text{-}Cl(A^{*\pi}) \subseteq A^{*\pi}$. Thus $A^{*\pi}(\mathcal{I}) = \pi\text{-}Cl(A^{*\pi}(\mathcal{I}))$.

<u>Claim:</u> $A^{*\pi} \subseteq \pi\text{-}Cl(A)$.
Let $x \in A^{*\pi} \Rightarrow U_x \cap A \neq \phi\ for\ every\ U_x \in \pi N(x)$. This shows that $x \in \pi\text{-}Cl(A)$.Thus $A^{*\pi} \subseteq \pi\text{-}Cl(A)$ .Hence $A^{*\pi}(\mathcal{I}) = \pi\text{-}Cl(A^{*\pi}(\mathcal{I})) \subseteq \pi\text{-}Cl(A)$.

(4) $(A\cup B)^{*\pi}(\mathcal{I}) = \{ x \in X / U_x \cap (A\cup B) \notin \mathcal{I}\ for\ every\ U_x \in \pi N(x)\}$
$= \{x \in X / (U_x \cap A) \cup (U_x \cap B) \notin \mathcal{I}\ for\ every\ U_x \in \pi N(x)\}$
$= \{x \in X / U_x \cap A \notin \mathcal{I}$ or $Ux \cap B \notin \mathcal{I}\ for\ every\ U_x \in \pi N(x)\}$
$= \{x \in X / U_x \cap A \notin \mathcal{I}\ for\ every\ U_x \in \pi N(x)\}$ or $\{x \in X / U_x \cap B \notin \mathcal{I}\ for\ every\ U_x \in \pi N(x)\}$
$= A^{*\pi}(\mathcal{I}) \cup B^{*\pi}(\mathcal{I})$.

(5) By (2) we know that $A \subseteq B \Rightarrow A^{*\pi} \subseteq B^{*\pi}$
$\Rightarrow (A\cap B)^{*\pi} \subseteq A^{*\pi}$ and $(A\cap B)^{*\pi} \subseteq B^{*\pi}$
$\Rightarrow (A\cap B)^{*\pi}(\mathcal{I}) \subseteq A^{*\pi}(\mathcal{I}) \cap B^{*\pi}(\mathcal{I})$

(6) $A^{*\pi}(\mathcal{I}) - B^{*\pi}(\mathcal{I}) = \{x\in X/ U_x \cap A \notin \mathcal{I}\ for\ every\ U_x \in \pi N(x)\} - \{x\in X/ U_x \cap B \notin \mathcal{I}\ for\ every\ U_x \in \pi N(x)\} = \{x\in X/ U_x \cap (A-B) \notin \mathcal{I}\ for\ every\ U_x \in \pi N(x)\} = (A - B)^{*\pi}(\mathcal{I})$.

(7) By (3) $A^{*\pi} \subseteq \pi\text{-}Cl(A)$. Replace A by $A^{*\pi}$. Then $(A^{*\pi})^{*\pi} \subseteq \pi\text{-}Cl(A^{*\pi}) = A^{*\pi}$

(8) Let $E \in \mathcal{I}$. $(A\cup E)^{*\pi} = A^{*\pi} \cup E^{*\pi} = A^{*\pi}$ [$\because E^{*\pi} = \phi$ by the definition]
$(A-E)^{*\pi} = \{x\in X/ U_x \cap (A-E) \notin \mathcal{I}\ for\ every\ U_x \in \pi N(x)\}$
$= \{x\in X/ U_x \cap A \notin \mathcal{I}\ for\ every\ U_x \in \pi N(x)\}$
$= A^{*\pi}$
Therefore $(A\cup E)^{*\pi}(\mathcal{I}) = A^{*\pi}(\mathcal{I}) = (A - E)^{*\pi}(\mathcal{I})$.

(9) Let $U \in \pi O(X)$.
$U\cap A^{*\pi} = U\cap \{x\in X/ U_x \cap A \notin I, for\ every\ U_x \in \pi N(x)\}$

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$= U\cap \{x\in X/\ U_x \cap (U\cap A) \notin \mathcal{I}, for\ every\ U_x \in \pi N(x)\}$
$= U\cap (U\cap A)^{*\pi}$
$\subseteq (U\cap A)^{*\pi}$

Hence $U\cap A^{*\pi}(\mathcal{I}) = U\cap (U\cap A)^{*\pi} \subseteq (U\cap A)^{*\pi}(\mathcal{I})$.

Corollary: 3.13

If $(X, \tau, \mathcal{I})$ is a space and $A \subseteq X$ then the following hold:
(1) $(A^{*\pi})^{*\pi} \subseteq A^{*\pi} = \pi\text{-Cl}(A^{*\pi}) \subseteq \pi\text{-Cl}(A)$.
(2) $(A^{*\pi})^{*\pi} \subseteq \pi\text{-Cl}(A^{*\pi}) \subseteq \pi\text{-Cl}(A)$
(3) $\pi\text{-Cl}(A^{*\pi}) = \text{Cl}(A^{*\pi})$ if $A^{*\pi}$ is dense
(4) $A^{*}\subseteq A^{*\pi}\subseteq \pi\text{-Cl}(A)$

Theorem: 3.14

Let $(X, \tau, \mathcal{I})$ be a space and $A \subseteq X$ then the following hold:
(1). $(X-E)^{*\pi} = X^{*\pi}$, if $E \in \mathcal{I}$
(2). $[X- (A-E)]^{*\pi} = [(X-A) \cup E]^{*\pi}$, $E \in \mathcal{I}$

Proof:

(1) Let $(X, \tau, \mathcal{I})$ be a space and $A \subseteq X$.
$x \in (X-E)^{*\pi} \Rightarrow \{ x\in X/\ U_x \cap (X-E) \notin \mathcal{I}, for\ every\ U_x \in \pi N(x)\}$
$\Rightarrow \{ x\in X/\ U_x \cap X)-(U_x\cap E) \notin \mathcal{I}, for\ every\ U_x \in \pi N(x)\}$
$\Rightarrow U_x \cap X \notin \mathcal{I}, for\ every\ U_x \in \pi N(x)$
$\Rightarrow x \in X^{*\pi}$
$\Rightarrow (X-E)^{*\pi} \subseteq X^{*\pi}$
$x \in X^{*\pi} \Rightarrow U_x \cap X \notin \mathcal{I}, for\ every\ U_x \in \pi N(x)$
$\Rightarrow U_x \cap X)- (U_x\cap E) \notin \mathcal{I}, for\ every\ U_x \in \pi N(x)$
$\Rightarrow U_x \cap (X-E) \notin \mathcal{I}, for\ every\ U_x \in \pi N(x)$
$\Rightarrow x \in (X-E)^{*\pi}$
$\Rightarrow X^{*\pi} \subseteq (X-E)^{*\pi}$
Therefore $(X-E)^{*\pi} = X^{*\pi}$, if $E \in \mathcal{I}$.
(2) If $E \in \mathcal{I}$ then $(A\cup E)^{*\pi} = A^{*\pi} = (A- E)^{*\pi}$ Replace A by $X-A$.
Then $[(X-A) \cup E]^{*\pi} = (X-A)^{*\pi} = (X- (A-E))^{*\pi}$
The following theorem gives some properties for the operator $\text{Cl}^{*\pi}$

Theorem: 3.15

For arbitrary subsets A and B of X we have
(1) $A \subseteq B$ implies $\text{Cl}^{*\pi}(A) \subseteq \text{Cl}^{*\pi}(B)$
(2) $\text{Cl}^{*\pi}(A\cup B) = \text{Cl}^{*\pi}(A) \cup \text{Cl}^{*\pi}(B)$
(3) $\text{Cl}^{*\pi}(A\cap B) \subseteq \text{Cl}^{*\pi}(A) \cap \text{Cl}^{*\pi}(B)$
(4) $\text{Cl}^{*\pi}(\text{Cl}^{*\pi}(A)) \subseteq \text{Cl}^{*\pi}(A)$

Theorem: 3.16

For any $A, B \subseteq X$, $\pi\text{-Cl}(A^{*\pi}) \cup \pi\text{-Cl}(B^{*\pi}) \subseteq \pi\text{-Cl}(A^{*\pi} \cup B^{*\pi})$

Proof:

$\pi\text{-Cl}(A^{*\pi}) \cup \pi\text{-Cl}(B^{*\pi}) = (A^{*\pi})^{*\pi} \cup (B^{*\pi})^{*\pi} \subseteq \pi\text{-Cl}(A^{*\pi}) \cup \pi\text{-Cl}(B^{*\pi}) = \pi\text{-Cl}((A^{*\pi}\cup B^{*\pi}))$.
Therefore $\pi\text{-Cl}(A^{*\pi}) \cup \pi\text{-Cl}(B^{*\pi}) \subseteq \pi\text{-Cl}(A^{*\pi} \cup B^{*\pi})$.

Theorem: 3.17

Let (X, τ) be a space and $\mathcal{I}$ and $\mathcal{J}$ are two ideals on X and let A be a subset of X then $A^{*\pi}(\mathcal{J}) \subseteq A^{*\pi}(\mathcal{I})$, if $\mathcal{I} \subseteq \mathcal{J}$.

Proof:

$A^{*\pi}(\mathcal{J}) = \{x\in X/\ U_x \cap A \notin \mathcal{J}, for\ every\ U_x \in \pi N(x)\}$
$\subseteq \{x\in X/\ U_x \cap A \notin \mathcal{I}, for\ every\ U_x \in \pi N(x)\} = A^{*\pi}(\mathcal{I})$.

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Hence $A^{*\pi}(\mathcal{J}) \subseteq A^{*\pi}(\mathcal{I})$.

Theorem: 3.18

If $\mathcal{I}$ and $\mathcal{J}$ are two ideals on (X, τ) such that $\mathcal{I} \subseteq \mathcal{J}$ then $\tau^{*\pi}(\mathcal{I}) \subseteq \tau^{*\pi}(\mathcal{J})$.

Proof:

Let $A \in \tau^{*\pi}(\mathcal{I}) \Rightarrow (X - A)$ is $\tau^{*\pi}(\mathcal{I})$ closed.

$$\Rightarrow Cl^{*\pi}(X - A) = (X - A)$$
$$\Rightarrow (X - A) \cup (X - A)^{*\pi} = (X - A)$$
$$\Rightarrow (X - A)^{*\pi}(\mathcal{I}) \subseteq (X - A)$$
$$\Rightarrow (X - A)^{*\pi}(\mathcal{J}) \subseteq (X - A)$$
$$\Rightarrow A \in \tau^{*\pi}(\mathcal{J})$$

Hence $\tau^{*\pi}(\mathcal{I}) \subseteq \tau^{*\pi}(\mathcal{J})$.

Theorem: 3.19

Let $(X, \tau, \mathcal{I})$ is a space and $A \subseteq X$ then $A^{*\pi} - (A^{*\pi})^{*\pi} \subseteq (A - A^{*\pi})^{*\pi}$

Proof:

Let $x \in A^{*\pi} - (A^{*\pi})^{*\pi}$. Then $x \in A^{*\pi}$. That is $U_x \cap A \notin \mathcal{I}, for\ every\ U_x \in \pi N(x)$. Thus $U_x \cap ((A - A^{*\pi})^{*\pi}) \notin \mathcal{I}, for\ every\ U_x \in \pi N(x)$. Therefore $x \in (A - A^{*\pi})^{*\pi}$. Hence $A^{*\pi} - (A^{*\pi})^{*\pi} \subseteq (A - A^{*\pi})^{*\pi}$.

Theorem: 3.20

Let (X, τ) be a space and $\mathcal{I}$ and $\mathcal{J}$ are two ideals on X and let A be a subset of X then $A^{*\pi}(\mathcal{I} \cap \mathcal{J}) = A^{*\pi}(\mathcal{I}) \cup A^{*\pi}(\mathcal{J})$ where $\mathcal{I} \cap \mathcal{J}$ is an ideal on X.

Proof:

Let (X, τ) be a space and $\mathcal{I}$ and $\mathcal{J}$ are two ideals on X and let A be a subset of X. Also $\mathcal{I} \cap \mathcal{J}$ is an ideal on X.

$A^{*\pi}(\mathcal{I} \cap \mathcal{J}) = \{x \in X / U_x \cap A \notin \mathcal{I} \cap \mathcal{J}, for\ every\ U_x \in \pi N(x)\}$

$= \{x \in X / U_x \cap A \notin \mathcal{I}$ or $U_x \cap A \notin \mathcal{J}, for\ every\ U_x \in \pi N(x)\}$

$= \{x \in X / U_x \cap A \notin \mathcal{I}\ for\ every\ U_x \in \pi N(x)\} \cup x \in X - U_x \cap A \notin \mathcal{J}$ for every $U_x \in \pi N(x)\}$

$= A^{*\pi}(\mathcal{I}) \cup A^{*\pi}(\mathcal{J})$.

Theorem: 3.21

Let (X, τ) be a space and $\mathcal{I}$ and $\mathcal{J}$ are two ideals on X and $A \subseteq X$ then $A^{*\pi}(\mathcal{I} \cup \mathcal{J}, \tau) = A^{*\pi}(\mathcal{I}, \tau^{*\pi}(\mathcal{J})) \cap A^{*\pi}(\mathcal{J}, \tau^{*\pi}(\mathcal{I}))$, where $\mathcal{I} \cup \mathcal{J} = \{E \cup H \backslash E \in \mathcal{I}$ and $H \in \mathcal{J}\}$ is an ideal on X.

Proof:

Let (X, τ) be a space and $\mathcal{I}$ and $\mathcal{J}$ are two ideals on X and $A \subseteq X$.

Suppose $x \notin A^{*\pi}(\mathcal{I} \cup \mathcal{J}, \tau)$.

Then there exists $U \in \pi N(x)$ such that $U \cap A \in (\mathcal{I} \cup \mathcal{J})$.

Let $E \in \mathcal{I}$ and $H \in \mathcal{J}$ such that $U \cap A = E \cup H$.

Without loss of generality we assume that $E \cap H = \phi$.

Thus $(U \cap A) - E = H$ and $(U \cap A) - H = E$.

This shows that $(U - E) \cap A = H \in \mathcal{J}$ and $(U - H) \cap A = E \in \mathcal{I}$.

$\Longrightarrow x \notin A^{*\pi}(\mathcal{J}, \tau^{*\pi}(\mathcal{I}))$ or $x \notin A^{*\pi}(\mathcal{I}, \tau^{*\pi}(\mathcal{J}))$.

Hence $A^{*\pi}(\mathcal{J}, \tau^{*\pi}(\mathcal{I})) \cap A^{*\pi}(\mathcal{I}, \tau^{*\pi}(\mathcal{J})) \subseteq A^{*\pi}(\mathcal{I} \cup \mathcal{J}, \tau)$ ------------------ (1)

To prove $A^{*\pi}(\mathcal{I} \cup \mathcal{J}, \tau) \subseteq A^{*\pi}(\mathcal{I}, \tau^{*\pi}(\mathcal{J}))$.

Let $x \notin A^{*\pi}(\mathcal{I}, \tau^{*\pi}(\mathcal{J}))$.

Then there exists $U \in \pi N(x) \supseteq U \cap A \in \mathcal{I}$ in $\tau^{*\pi}(\mathcal{J})$.

Therefore $U \cap A = E$ for $E \in \mathcal{I}$ implies that $(U - E) \cap A \in \mathcal{I}$.

Hence $U \cap A = E \cup H \in (\mathcal{I} \cup \mathcal{J}, \tau)$.

This shows that $x \notin A^{*\pi}(\mathcal{I} \cup \mathcal{J}, \tau)$.

Thus $A^{*\pi}(\mathcal{I} \cup \mathcal{J}, \tau) \subseteq A^{*\pi}(\mathcal{I}, \tau^{*\pi}(\mathcal{J}))$ -----------------(2)

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Similarly $A^{*\pi}(\mathcal{I} \cup \mathcal{J}, \tau) \subseteq A^{*\pi}(\mathcal{J}, \tau^{*\pi}(\mathcal{I}))$ -------------------(3)
From (2) and (3), $A^{*\pi}(\mathcal{I} \cup \mathcal{J}, \tau) \subseteq A^{*\pi}(\mathcal{I}, \tau^{*\pi}(\mathcal{J})) \cap A^{*\pi}(\mathcal{J}, \tau^{*\pi}(\mathcal{I}))$ ---------------(4)
From (1) and (4), $A^{*\pi}(\mathcal{I} \cup \mathcal{J}, \tau) = A^{*\pi}(\mathcal{I}, \tau^{*\pi}(\mathcal{J})) \cap A^{*\pi}(\mathcal{J}, \tau^{*\pi}(\mathcal{I}))$.

Theorem: 3.22

The following are equivalent for a subset $A \subseteq (X, \tau, \mathcal{I})$.

(1) $A \in \tau^{*\pi}$
(2) $(X - A)$ is $\tau^{*\pi}$ closed
(3) $(X - A^{*\pi}) \subseteq (X - A)$
(4) $A \subseteq X - (X - A)^{*\pi}$

4. THE SET OPERATOR $\Psi^{\pi}(\mathcal{I},\tau)$

Definition: 4.1

Let $(X, \tau, \mathcal{I})$ be a space, a set operator $\Psi^{\pi}(\mathcal{I},\tau) : P(X)\to\tau$ is defined as follows: For every $A\subseteq X$, $\Psi^{\pi}(\mathcal{I},\tau) : \{x \in X$ / there exists a $U \in \pi N(X)$ such that $U - A \in \mathcal{I}\}$. We denote $\Psi^{\pi}(\mathcal{I},\tau)$ simply by Ψ^{π}.

Lemma: 4.2

The operator $\Psi^{\pi}(\mathcal{I},\tau)$ is a natural complement to the operator $()^{*\pi}$.

Proof:

To prove $\Psi^{\pi}(\mathcal{I},\tau) = X - (X-A)^{*\pi}$.

Let $x \in \Psi^{\pi}(A) \Leftrightarrow$ there exists $U \in \pi N(X)$ such that $U - A \in \mathcal{I}\}$
$\Leftrightarrow$ there exists $U \in \pi N(X)$ such that $U \cap (U - A) \in \mathcal{I}\}$
$\Leftrightarrow x \notin (X-A)^{*\pi}$
$\Leftrightarrow x \in X- (X-A)^{*\pi}$

Hence $\Psi^{\pi}(\mathcal{I},\tau) = X - (X-A)^{*\pi}$.

Therefore the operator $\Psi^{\pi}(\mathcal{I},\tau)$ is a natural complement to the operator $()^{*\pi}$.

Corollary: 4.3

For $A \subseteq (X, \tau, \mathcal{I})$, we have (1) If $\mathcal{I} = \{\phi\}$ then $\Psi^{\pi}(A) = \pi\text{-int}(A)$
(2) If $\mathcal{I} = P(X)$ then $\Psi^{\pi}(A) = X$

Lemma: 4.4

$\Psi^{\pi}(A) \subseteq \Psi(A)$ for every $A \subseteq (X, \tau, \mathcal{I})$.

Remark: 4.5

The converse of the above need not be true.

Example: 4.6

$X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\phi, \{a\}\}$. Let $A = \{a, b, c\}$. Then $X - A = \{d\}$ and $(X-A)^{*} = \{d\}$. Therefore $\Psi(A) = X - (X-A)^{*} = \{a, b, c\}$ --------------- (1)
Now $(X-A)^{*\pi} = \{c, d\}$.Then $\Psi^{\pi}(A) = X - (X-A)^{*\pi} = \{a, b\}$---------------------(2)
Hence from (1) and (2) we have $\Psi^{\pi}(A) \not\supseteq \Psi(A)$.

Theorem: 4.7

Let $(X, \tau, \mathcal{I})$ be a ideal space and $A, B \subseteq P(X)$. Then

(1) If $A \subseteq B$ then $\Psi^{\pi}(A) \subseteq \Psi^{\pi}(B)$
(2) If $A, B \in P(X)$ then $\Psi^{\pi}(A\cap B) \subseteq \Psi^{\pi}(A) \cap \Psi^{\pi}(B)$
(3) If $U \in \tau^{*\pi}(\mathcal{I})$ then $U \subseteq \Psi^{\pi}(U)$
(4) If $A \in P(X)$ then $\Psi^{\pi}(A) \subseteq \Psi^{\pi}(\Psi^{\pi}(A))$
(5) If $A \in P(X)$ then $\Psi^{\pi}(A) = \Psi^{\pi}(\Psi^{\pi}(A)) \Leftrightarrow (X-A)^{*\pi} = ((X-A)^{*\pi})^{*\pi}$

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(6) If $A \in \mathcal{I}$ then $\Psi^{\pi}(A) = X - X^{*\pi}$
(7) If $A \in P(X)$, $E \in \mathcal{I}$ then $\Psi^{\pi}(A-E) = \Psi^{\pi}(A)$
(8) If $A \in P(X)$, $E \in \mathcal{I}$ then $\Psi^{\pi}(A\cup E) = \Psi^{\pi}(A)$
(9) If $(A - B) \cup (B-A) \in \mathcal{I}$ then $\Psi^{\pi}(A) = \Psi^{\pi}(B)$

Corollary: 4.8

If $(X, \tau, \mathcal{I})$ is a ideal space, $A \subseteq X$ and $E \in \mathcal{I}$ then $\Psi^{\pi}(A-E) = \Psi^{\pi}(A \cup E) = \Psi^{\pi}(A)$.

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