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INTUITIONISTIC FUZZY SIMILARITY MEASURE BASED ON TANGENT FUNCTION AND ITS APPLICATION TO MULTI-ATTRIBUTE DECISION MAKING

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ABSTRACT
In this paper, the tangent similarity measure of intuitionistic fuzzy sets is proposed and some of its properties are studied. The concept of the tangent similarity measure of intuitionistic fuzzy sets is a decision making tool which is characterized by the degree of membership function, degree of non-membership function and degree of hesitancy (sum of this three components is equal to one). Finally, using this proposed approach, an application on medical diagnosis is provided to show the applicability and effectiveness of the proposed approach.

General Terms
Similarity measure, Intuitionistic fuzzy sets

Keywords
Tangent similarity measure, Degree of hesitancy, Decision making, Medical diagnosis

1. INTRODUCTION
Fuzzy sets, a generalization of crisp sets, was proposed by Zadeh [30] in order to deal with uncertainty. Research on the theory of fuzzy sets has been growing steadily since the inception of the theory in 1965. Literature review reveals that research on a broad variety of applications has also been very active and various generalizations of fuzzy sets have been proposed. In fuzzy set theory, membership and non-membership degrees are complementary, i.e., the sum of both degrees of an element belonging in a fuzzy set is equal to one. However, there exist some situations where the membership and non membership degrees are not complementary. In order to deal such situations intuitionistic fuzzy set (IFS) was introduced by Atanassov [1]. For intuitionistic fuzzy set, each element is assigned by membership and non-membership degrees, where the sum of the two degrees does not exceed one. So there must have an attention to study the intuitionistic fuzzy theory. The concept of IFS has been widely studied and applied in various areas such as decision-making problems [16, 22, 26, 27], educational problem [19], selection problem [15], medical diagnosis [2, 11, 15, 23],

pattern recognition [3, 5, 7, 9, 24] and so on. Similarity measure is an important tool for determining the degree of similarity between two objects. Kaufman and Rousseeuw [10] presented some applications of traditional similarity measures in hierarchical cluster analysis.

Similarity measures between intuitionistic fuzzy sets have been widely studied in both theoretical and practical aspects for last two decades. In 2001, Hung and Yang [4] adopted the Hausdorff distance to develop several similarity measures. In 2002, Dengfeng and Chuntian [3] presented the axiomatic definition of similarity measures between intuitionistic fuzzy sets and proposed similarity measures based on high membership and low membership functions. In 2003, Liang and Shi [13] demonstrated some counter-intuitive cases resulting from the measures in [3] and then presented several similarity measures to overcome those cases. In 2003, Mitchell [14] studied similarity measures on IFSs from the statistical point of view to modify Dengfeng and Chuntian's methods [3]. In 2007, Rezaei and Mukaidono [20] reviewed existing similarity measures of IFSs and showed that not all existing similarity measures are effective and reasonable in some cases.

In 2007, Xu [26] introduced the concepts of positive and negative ideal IFS and extended some similarity measures by assigning weights. Xu [26] applied these similarity measures to solve multi-attribute decision making problems. In 2007, Khatibi and Montazer [11] studied similarity measures. They conducted experiments for bacterial classification using three similarity measures: one for fuzzy sets, and two for intuitionistic fuzzy sets. The obtained results from the experiments reflected that the both measures for IFSs outperformed case for the fuzzy sets. In 2008, Xu and Chen [28] presented a comprehensive overview of distance and similarity measures of IFSs where they also extended those existing measures based on a geometric distance model. In 2011, Ye [29] developed cosine and weighted cosine similarity measures for IFSs and applied these concepts to medical diagnosis

In 2012, Hwang et al. [7] proposed a new similarity measure for IFSs based on Sugeno integral [21] and embedded the measure in a robust clustering algorithm for pattern recognition. They illustrated some examples to provide comparisons between the proposed similarity measure and several existing methods [3, 4, 6, 13, 14]. Numerical results evidenced that their proposed similarity measure [7] is more reasonable than those previous methods.

In 2007, Li et al. [12] and in 20013, Papakostas et al. [17] studied comparative analysis of similarity measures providing theoretical and computational aspects of similarity measures in the literature. In 2014, Intarapaiboon [8] proposed two new similarity measures for IFSs and applied to pattern recognition.

Although many similarity measures of IFSs have been proposed, no studies have considered the tangent similarity measure. Recently, Pramanik and Mondal [18] proposed weighted fuzzy similarity measure based on tangent function and its application to medical diagnosis. In this paper the concept of fuzzy tangent similarity measure is extended to intuitionistic tangent similarity measure and studied some of its properties.

In this paper, we present new similarity measure called 'intuitionistic fuzzy tangent similarity measures'. The rest of the paper proceeds as follow: Section2 presents the concepts of IFSs and tangent similarity measures for intuitionistic fuzzy sets. Section3 presents decision making method based on tangent similarity measures. Section 4 is devoted to present application of the proposed tangent similarity measure to medical diagnosis. Finally, Section 5 presents the conclusion and future scope of research.

2. MATHEMATICAL PRELIMINARIES

2.1 Intuitionistic fuzzy sets

In 1965, Zadeh [30] introduced the concept of fuzzy sets as a mathematical form for representing impreciseness.

Definition 2.1: Fuzzy set: A fuzzy set A in a universe of discourse X is defined as the following set of pairs $A=\{(x,\mu_A(x)): x\in X\}$. Here, $\mu_A(x): x\to[0,1]$ is a mapping called the membership value of x ∈ X in a fuzzy set A

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Definition 2.2: Intuitionistic Fuzzy set: An intuitionistic fuzzy set (IFS) [1] $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$, where the functions $\mu_A(x) : x \to [0,1]$ and $\nu_A(x) : x \to [0,1]$ define the degree of membership and degree of non-membership respectively of the element x ∈ X to the set A that is a subset of X, and every x ∈ X, $0 \le \mu_A(x) + \nu_A(x) \le 1$.

Definition 2.3: The value of $\xi_A(x) = 1 - (\mu_A(x) + \nu_A(x))$ is called the degree of non determinacy (or uncertainty or hesitancy) of the element x ∈ X to the intuitionistic fuzzy set.

Definition 2.4: Hamming distance is defined as H (A, B) = $\frac{1}{2}\sum_{i=1}^{n} |\mu_A(x) - \mu_B(x)| + |\nu_A(x) - \nu_B(x)| + |\xi_A(x) - \xi_B(x)|$

Definition 2.5: Proposed tangent similarity measure for intuitionistic fuzzy sets

Let $P = (\mu_P(x_i), \nu_P(x_i), \xi_P(x_i))$ and $Q = (\mu_Q(x_i), \nu_Q(x_i), \xi_Q(x_i))$ be two intuitionistic fuzzy numbers. Now we present tangent similarity function which measures the similarity between two vectors based only on the direction, ignoring the impact of the distance between them can be presented as:

$$T_{IFS}(P, Q) = \frac{1}{n}\sum_{i=1}^{n}\left\langle 1 - \tan\left(\frac{\pi\left(|\mu_P(x_i) - \mu_Q(x_i)| + |\nu_P(x_i) - \nu_Q(x_i)| + |\xi_P(x_i) - \xi_Q(x_i)|\right)}{12}\right)\right\rangle \qquad (1)$$

Propositions

The defined tangent similarity measure $T_{IFS}(P, Q)$ between two intuitionistic fuzzy numbers P and Q satisfies the following properties:

1. $0 \le T_{IFS}(P, Q) \le 1$
2. $T_{IFS}(P, Q) = 1$ if and only if $P = Q$
3. $T_{IFS}(P, Q) = T_{IFS}(Q, P)$
4. If R is a IFS in X and $P \subset Q \subset R$ then

$T_{IFS}(P, R) \le T_{IFS}(P, Q)$ and $T_{IFS}(P, R) \le T_{IFS}(Q, R)$

Proofs:

(**1**)

As the membership, non-membership and hesitancy functions of the intuitionistic fuzzy set are in [0, 1] and the value of the tangent function are within [0 ,1], the similarity measure based on tangent function also is within [0,1]. Hence $0 \le T_{IFS}(P, Q) \le 1$

(**2**)

For any two intuitionistic fuzzy sets P and Q if $P = Q$ this implies $\mu_P(x_i) = \mu_Q(x_i), \nu_P(x_i) = \nu_P(x_i), \xi_P(x_i) = \xi_P(x_i)$.

Hence $|\mu_P(x_i) - \mu_Q(x_i)| = 0$, $|\nu_P(x_i) - \nu_Q(x_i)| = 0$, $|\xi_P(x_i) - \xi_Q(x_i)| = 0$, Thus $T_{IFS}(P, Q) = 1$

Conversely,

If $T_{IFS}(P, Q) = 1$ then $|\mu_P(x_i) - \mu_Q(x_i)| = 0$, $|\nu_P(x_i) - \nu_Q(x_i)| = 0$, $|\xi_P(x_i) - \xi_Q(x_i)| = 0$ since tan(0)=0.

So we can write $\mu_P(x_i) = \mu_Q(x_i), \nu_P(x_i) = \nu_Q(x_i), \xi_P(x_i) = \xi_Q(x_i)$. Hence P = Q.

(3)

This proof is obvious.

(4)

If $P \subset Q \subset R$ then $T_P(x_i) \leq T_Q(x_i) \leq T_R(x_i)$, $I_P(x_i) \geq I_Q(x_i) \geq I_R(x_i)$, $F_P(x_i) \geq F_Q(x_i) \geq F_R(x_i)$ for $x_i \in X$.

Now we have the following inequalities:

$$|T_P(x_i) - T_Q(x_i)| \leq |T_P(x_i) - T_R(x_i)|,$$

$$|T_Q(x_i) - T_R(x_i)| \leq |T_P(x_i) - T_R(x_i)|;$$

$$|I_P(x_i) - I_Q(x_i)| \leq |I_P(x_i) - I_R(x_i)|,$$

$$|I_Q(x_i) - I_R(x_i)| \leq |I_P(x_i) - I_R(x_i)|;$$

$$|F_P(x_i) - F_Q(x_i)| \leq |F_P(x_i) - F_R(x_i)|,$$

$$|F_Q(x_i) - F_R(x_i)| \leq |F_P(x_i) - F_R(x_i)|.$$

Thus $T_{IFS}(P, R) \leq T_{IFS}(P, Q)$ and $T_{IFS}(P, R) \leq T_{IFS}(Q, R)$. Since tangent function is increasing in the interval $\left[0, \frac{\pi}{4}\right]$.

3. INTUITIONISTIC FUZZY DECISION MAKING BASED ON TANGENT SIMILARITY MEASURES

Let $A_1, A_2, ..., A_m$ be a discrete set of candidates, $C_1, C_2, ..., C_n$ be the set of criteria of each candidate, and $B_1, B_2, ..., B_k$ are the alternatives of each candidates. The decision-maker provides the ranking of alternatives with respect to each candidate. The ranking presents the performances of candidates A_i (i = 1, 2,..., m) against the criteria C_j (j = 1, 2, ..., n). The values associated with the alternatives for MADM problem can be presented in the following decision matrix (see Table 1).

Table 1: The relation between candidates and attributes

	C_1	C_2	...	C_n
A_1	$[AC]_{11}$	$[AC]_{12}$	...	$[AC]_{1n}$
A_2	$[AC]_{21}$	$[AC]_{22}$	...	$[AC]_{2n}$
...	...	...	...	...
A_m	$[AC]_{m1}$	$[AC]_{m2}$	...	$[AC]_{mn}$

Table 2: The relation between attributes and alternatives

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	B_1	B_2	...	B_k
C_1	$[CB]_{11}$	$[CB]_{12}$	...	$[CB]_{1k}$
C_2	$[CB]_{21}$	$[CB]_{22}$	...	$[CB]_{2k}$
...	...	...	...	...
C_n	$[CB]_{n1}$	$[CB]_{n2}$	...	$[CB]_{nk}$

Here $[AC]_{ij}$ and $[CB]_{ij}$ and are all intuitionistic fuzzy numbers.

The steps of decision making corresponding to intuitionistic fuzzy number based on tangent function are presented as follows.

Step 1: Determination of the relation between candidates and attributes

Each candidate A_i (i = 1, 2, ..., m) having the attribute C_j (j = 1, 2, ..., n) is presented as follows:

Table 3: Relation between candidates and attributes in terms of intuitionistic fuzzy numbers

	C_1	C_2	...	C_n
A_1	$\langle T_{11}, I_{11}, F_{11}\rangle$	$\langle T_{12}, I_{12}, F_{12}\rangle$	...	$\langle T_{1n}, I_{1n}, F_{1n}\rangle$
A_2	$\langle T_{21}, I_{21}, F_{21}\rangle$	$\langle T_{22}, I_{22}, F_{22}\rangle$	...	$\langle T_{2n}, I_{2n}, F_{2n}\rangle$
...	...	...	...	...
A_m	$\langle T_{m1}, I_{m1}, F_{1m1}\rangle$	$\langle T_{m2}, I_{m2}, F_{m2}\rangle$	...	$\langle T_{mn}, I_{mn}, F_{mn}\rangle$

Step 2: Determination of the relation between attributes and alternatives

The relation between attributes C_i (i = 1, 2, ..., n) and alternatives B_t (t = 1, 2, ..., k) is presented as follows:

Table4: The relation between attributes and alternatives in terms of intuitionistic fuzzy numbers

	B_1	B_2	...	B_k
C_1	$\langle T_{11}, I_{11}, F_{11}\rangle$	$\langle T_{12}, I_{12}, F_{12}\rangle$	...	$\langle T_{1k}, I_{1k}, F_{1k}\rangle$
C_2	$\langle T_{21}, I_{21}, F_{21}\rangle$	$\langle T_{22}, I_{22}, F_{22}\rangle$	...	$\langle T_{2k}, I_{2k}, F_{2k}\rangle$
...	...	...	...	...
C_n	$\langle T_{n1}, I_{n1}, F_{n1}\rangle$	$\langle T_{n2}, I_{n2}, F_{n2}\rangle$	...	$\langle T_{nk}, I_{nk}, F_{nk}\rangle$

Step 3: Determination of the correlation measures between candidates and alternatives

Determine the correlation measure between table3 and table 4 using $T_{IFS}(P, Q)$ (from equation 1).

Step 4: Ranking the alternatives:

Rank the alternatives corresponding to each candidate is prepare as the descending order of correlation measures. Highest value indicates the best alternative for corresponding candidate.

Step 5: End

4. A NUMERICAL EXAMPLE ON MEDICAL DIAGNOSIS USING TANGENT SIMILARITY MEASURE

Let us consider an illustrative example adopted from Szmidt and Kacprzyk [22] with minor changes. Medical diagnosis consists of uncertainties and increased volume of information available to physicians from new medical technologies. The process of classifying different set of symptoms under a single name of a disease is a difficult task. In some practical situations, symptoms are characterized by three components namely, degree of membership, degree of non-membership and degree of hesitancy. The proposed similarity measure among the patients versus symptoms and symptoms versus diseases will provide proper medical diagnosis. The main feature of the proposed approach is that it considers membership, non-membership and hesitancy degree by taking one time inspection for diagnosis.

Now, an example of a medical diagnosis will be presented. Example: Let P = {P_1, P_2, P_3, P_4} be a set of patients, D = {Viral fever, Malaria, Typhoid, Stomach problem, Chest problem} be a set of diseases and S = {Temperature, Headache, Stomach pain, cough, Chest pain.} be a set of symptoms. Our strategy is to examine the patient which will provide us membership, non-membership and hesitancy function for each patient. The relation between patients and symptoms is presented in the table 5). The relation between symptoms and diseases is presented in the table 6. The correlation measure between the above-mentioned two relations is shown in the table 7.

Step 1: Determination of the relation between candidates and attributes

The relation between patients and symptoms based on expert's opinion is presented in table 5.

Table 5: (Relation-1) - The relation between patient and symptoms

Relation-1	Temperature	Headache	Stomach pain	Cough	Chest pain
P_1	(0.8, 0.1, 0.1)	(0.6, 0.1, 0.3)	(0.2, 0.8, 0.0)	(0.6, 0.1, 0.3)	(0.1,0.6, 0.3)
P_2	(0.0, 0.8, 0.2)	(0.4, 0.4, 0.2,)	(0.6, 0.1, 0.3)	(0.1, 0.7, 0.2)	(0.1, 0.8, 0.1)
P_3	(0.8, 0.1, 0.1)	(0.8, 0.1, 0.1)	(0.0, 0.6, 0.4)	(0.2, 0.7, 0.1)	(0.0, 0.5, 0.5)
P_4	(06, 0.1, 0.3)	(0.5, 0.4, 0.1)	(0.3, 0.4, 0.3)	(0.7, 0.2, 0.1)	(0.3, 0.4, 0.3)

Step 2: Determination of the relation between attributes and alternatives

The relation between symptoms and diseases based on expert's opinion is presented in table 6.

Table 6: (Relation-2) -The relation among Symptoms and Diseases

Relation-2	Viral Fever	Malaria	Typhoid	Stomach problem	Chest problem
Temperature	(0.4, 0.0, 0.6)	(0.7, 0.0, 0.3)	(0.3, 0.3, 0.4)	(0.1, 0.7, 0.2)	(0.1, 0.8, 0.1)
Headache	(0.3,0.5,0.2)	(0.2, 0.6, 0.2)	(0.6, 0.1, 0.3)	(0.2, 0.4, 0.4)	(0.0, 0.8, 0.2)
Stomach pain	(0.1, 0.7, 0.2)	(0.0, 0.9, 0.1)	(0.2, 0.7, 0.1)	(0.8, 0.0, 0.2)	(0.2, 0.8, 0.0)
Cough	(0.4, 0.3, 0.3)	(0.7, 0.0, 0.3)	(0.2, 0.6, 0.2)	(0.2, 0.7, 0.1)	(0.2, 0.8, 0.0)
Chest pain	(0.1, 0.7, 0.2)	(0.1, 0.8, 0.1)	(0.1, 0.9, 0.0)	(0.2, 0.7, 0.1)	(0.8, 0.1, 0.1)

Step 3: Determination of the correlation measures between candidates and alternatives

The correlation measures between table5 and table 6 using $T_{IFS}(P, Q)$ (from equation 1) can be presented as follows (see table 7).

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Table 7: The correlation measure between Relation-1 and Relation-2

Tangent similarity measure	Viral Fever	Malaria	Typhoid	Stomach problem	Chest problem
P_1	0.85216	**0.87286**	0.86658	0.69458	0.69288
P_2	0.77070	0.72572	0.82878	**0.92652**	0.77242
P_3	0.79758	0.76302	**0.82962**	0.72668	0.69214
P_4	**0.84688**	0.84070	0.79776	0.76446	0.69672

Step 4: Ranking the alternatives:

The highest correlation measure (see the Table 7) reflects proper medical diagnosis. Therefore, patient P_1 suffers from Malaria, P_2 suffers from Stomach problem, P_3 suffers from Typhoid and P_4 suffers from Viral fever.

5. CONCLUSION

In this paper, we have proposed a tangent similarity measure of intuitionistic fuzzy sets and proved some of their basic properties. We have presented an application of tangent similarity measure of intuitionistic fuzzy sets in medical diagnosis problem. In the future work, the concept of tangent similarity measure can be extended to neutrosophic environment using singled valued neutrosophic sets [25].

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