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SYNTHESIS OF PLANAR MECHANISMS, PART III: FOUR-BAR MECHANISMS FOR THREE COUPLER-POSITIONS GENERATION

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ABSTRACT
Four-bar planar mechanisms have wide applications in industry and thus receive more attention from machinery design researchers.

The proposed approach in this paper relies on forming a mathematical model for the mechanism position incorporating the 3 coupler positions. The model consists of 8 nonlinear equations considering the transmission angle of the mechanism in the 3 coupler positions. A case study is presented as a justification for the proposed approach. Exact coupler positions are attained with transmission angles not more than 18 % of the optimum value of 90 degrees.

General Terms
Kinematics of mechanisms, mechanism synthesis, coupler positions generation.

Keywords
Synthesis of planar mechanisms, four-bar mechanisms, 3 coupler positions generation, nonlinear synthesis equations.

1. INTRODUCTION
Mechanism synthesis techniques range from simple graphical techniques going through analytical approaches with many assumptions and trials to sophisticated techniques using optimization application.

The subject of mechanism synthesis has occupied the attention of researchers over decades. Some publications are reviewed over the last 15 years to highlight some of the efforts focused on mechanism synthesis. Vujic and Radojkovic (2000) presented a procedure of a numerical method for kinematic synthesis of planar bar linkages in two or three infinitesimally close positions [1]. Saggere and Kota (2001) introduced a compliant-segment motion generation task where the coupler was a flexible segment and required a prescribed shape change along with a rigid-body motion [2].

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Cabrero, Simon and Prado (2002) set solution methods of optimal synthesis of planar mechanisms using a searching procedure through the application of genetic algorithms based on evolution techniques. They tested their method using the problem of four-bar mechanism synthesis [3]. Lebedov (2003) developed a vector method for the analysis of guidance and transmission mechanisms applied to 4-bar mechanisms [4].

Bulatovic and Djordjevic (2004) considered optimal synthesis of a four-bar linkage using the method of controlled deviations. They used the Hooke-Jeeves optimization algorithm which did not depend on the initial selection of mechanism dimensions [5]. Wu and Chen (2005) used an adjustable link to synthesize exactly any input-output relationship using a planar 4-bar mechanism [6]. Su and McCathy (2006) solved the synthesis equations for a complaint four-bar linkage with three specified equilibrium configurations. They used the polynomial homotopy continuation and the Newton-Raphson technique to assign the design candidates [7].

Hongying Dewei and Zhixing (2007) presented a computerized method using coupler-angle function curve to approximately synthesize a 4-bar path mechanism [8]. Sheu, Hu and Lee (2008) investigated the synthesis of a 4-bar mechanism with rolling contacts for motion and function generation [9]. Al-Smadi, Russell, Lee and Sodhi (2009) considered planar four-bar path generation with coupler point load, crank static torque, crank transverse deflection and follower buckling. They used the sequential quadratic programming algorithm to solve the nonlinear optimization problem of the mechanism synthesis [10]. Parlaktas , Soylemez and Tanik (2010) presented a novel method for the analysis and design of a certain type of geared four-bar mechanism with collinear input and output shafts [11]. Soong and Chang (2011) proposed a technique for the exact function generation problems of four-bar linkages using variable length driving links [12].

Kim and Yoo (2012) applied a unified synthesis approach to planar four-bar mechanisms for the purpose of function generation [13]. Tong (2013) developed techniques for the synthesis of planar four-bar linkages for tasks common to pick-and-place devices. He covered motion generation and path-point generation tasks. He used the geometric constraint programming and numerical solution of the synthesis equations [14]. Zhao, Yan and Ye (2014) focused on the synthesis of a flapping wing robot proposing a unified design formula for planar four-bar linkages with arbitrary n prescribed positions [15]. Hassaan (2015) presented an approach for the synthesis of three types of planar mechanisms fulfilling the requirements of specific stroke, time ratio and transmission angle. He formulated the synthesis equations in the form of nonlinear equations solved by MATLAB using the command 'fsolve' [16].

2. METHODOLOGY

The proposed methodology is applied to standard 4-bar mechanisms having fixed lengths. The approach is applied as follows:

- The desired 3 positions of the coupler are assigned in the motion plane.
- Closed loops are formed for the mechanism in the 3 positions.
- 2 equations are written for each loop in the x and y directions.
- 3 equations are written for the 3 transmission angles (one per mechanism position).
- The 6 equations are written in a normalized form by dividing each link dimension by r_2.
- The equations are written such that the right hand side is zero.
- The model in its final form consists of 6 nonlinear equations in 6 unknowns.
- The model is solved using MATLAB for the mechanism unknowns.

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Requirements:

It is desired to have a coupler of a known length in 3 positions: A_1B_1 , A_2B_2 and A_3B_3 with known orientations θ_{31} , θ_{32} and θ_{33} (Fig.1).

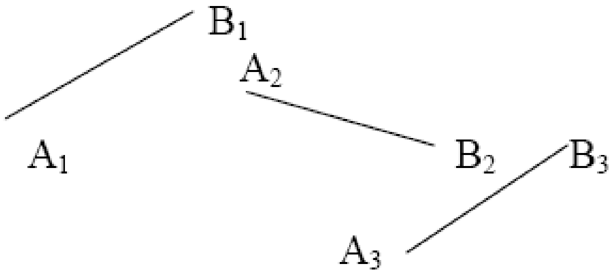

Fig.1: Mechanism coupler in the 3 positions.

Mechanism:

- Fig..2 shows a 4-bar mechanism in three positions according to the coupler three independent positions.

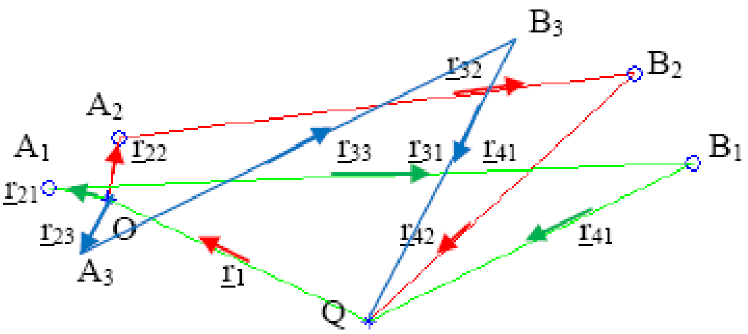

Fig.2: Four-bar mechanism in the 3 positions.

- Three polygons are closed which are required for displacement analysis in each mechanism position.

Analysis:

- The three coupler positions are: A_1B_1 , A_2B_2 and A_3B_3.
- Polygon 1: OA_1B_1QO. The displacement equation across the polygon is:

  $\underline{r}_1 + \underline{r}_{21} + \underline{r}_{31} + \underline{r}_{41} = 0$
- Considering the vectors components in the x-direction ; $\sum r_x = 0$ gives:

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$$r_1\cos\theta_1 + r_2 \cos\theta_{21} + r_3 \cos\theta_{31} + r_4 \cos\theta_{41} = 0 \quad (1)$$

- Considering the vectors components in the y-direction ; $\sum r_y = 0$ gives:

$$r_1\sin\theta_1 + r_2 \sin\theta_{21} + r_3 \sin\theta_{31} + r_4 \sin\theta_{41} = 0 \quad (2)$$

- Polygon 2: OA_2B_2QO. The displacement equation across the polygon is:

$$\underline{r}_1 + \underline{r}_{22} + \underline{r}_{32} + \underline{r}_{42} = 0$$

- Considering the vectors components in the x-direction ; $\sum r_x = 0$ gives:

$$r_1\cos\theta_1 + r_2 \cos\theta_{22} + r_3 \cos\theta_{32} + r_4 \cos\theta_{42} = 0 \quad (3)$$

- Considering the vectors components in the y-direction ; $\sum r_y = 0$ gives:

$$r_1\sin\theta_1 + r_2 \sin\theta_{22} + r_3 \sin\theta_{32} + r_4 \sin\theta_{42} = 0 \quad (4)$$

- Polygon 3: OA_3B_3QO. The displacement equation across the polygon is:

$$\underline{r}_1 + \underline{r}_{23} + \underline{r}_{33} + \underline{r}_{43} = 0$$

- Considering the vectors components in the x-direction ; $\sum r_x = 0$ gives:

$$r_1\cos\theta_1 + r_2 \cos\theta_{23} + r_3 \cos\theta_{33} + r_4 \cos\theta_{43} = 0 \quad (5)$$

- Considering the vectors components in the y-direction ; $\sum r_y = 0$ gives:

$$r_1\sin\theta_1 + r_2 \sin\theta_{23} + r_3 \sin\theta_{33} + r_4 \sin\theta_{43} = 0 \quad (6)$$

- Unknowns in Eqs.1-6: r_1, r_2, , r_4, θ_1, θ_{21}, θ_{41}, θ_{22} , θ_{42} , θ_{23} and θ_{43} .
- Number of unknowns: 10.
- Number of equations so far: 6.
- The number of design parameters is reduced through:
  1. Assigning the ground length, r_1.
  2. Using normalized dimensions by referring all the dimensions to r_2.
- In this case, the unknown design parameters are: $x_1 = r_{4n}$, $x_2 = \theta_1$, $x_3 = \theta_{21}$, $x_4 = \theta_{41}$, $x_5 = \theta_{22}$, $x_6 = \theta_{42}$, $x7 = \theta_{23}$ and $x8 = \theta_{43}$.
- Number of unknowns is reduced to 8.
- Two more equations may be written for the transmission angle in 2 of the 3 positions of the mechanism.
- The transmission angle is related to links 3 and 4 orientation angles through:

$$\mu_1 = \theta_{41} - \pi - \theta_{31}$$

and

$$\mu_2 = \theta_{42} - \pi - \theta_{32}$$

- Now, equations: 1 – 6 are written using the normalized dimensions and in a form suitable for MATLAB application as:

$$f_1 = r_{1n}\cos x_2 + \cos x_3 + r_{3n}\cos \theta_{31} + x_1\cos x_4 \quad (7)$$
$$f_2 = r_{1n}\sin x_2 + \sin x_3 + r_{3n}\sin \theta_{31} + x_1\sin x_4 \quad (8)$$
$$f_3 = r_{1n}\cos x_2 + \cos x_5 + r_{3n}\cos \theta_{32} + x_1\cos x_6 \quad (9)$$
$$f_4 = r_{1n}\sin x_2 + \sin x_5 + r_{3n}\sin \theta_{32} + x_1\sin x_6 \quad (10)$$
$$f_5 = r_{1n}\cos x_2 + \cos x_7 + r_{3n}\cos \theta_{32} + x_1\cos x_8 \quad (11)$$
$$f_6 = r_{1n}\sin x_2 + \sin x_7 + r_{3n}\sin \theta_{32} + x_1\sin x_8 \quad (12)$$
$$f_7 = \mu_1 - x_4 + \pi + \theta_{31} \quad (13)$$
$$f_8 = \mu_2 - x_6 + \pi + \theta_{32} \quad (14)$$

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Mechanism Synthesis:

- The synthesis equations are equations 7-14 (8 equations).
- The equations are nonlinear in 8 unknowns.
- The 8 equations are in the form: $f = 0$
- The 8 equations may be solved using the MATLAB command "fsolve" or any other numerical technique.
- The coordinates of B_1 will be use to locate the fixed pivot Q in the xy-plane.

Case Study:

It is required to design a 4-bar planar mechanism to move the coupler AB from position A_1B_1 to A_2B_2 to A_3B_3 as shown in Fig.3.

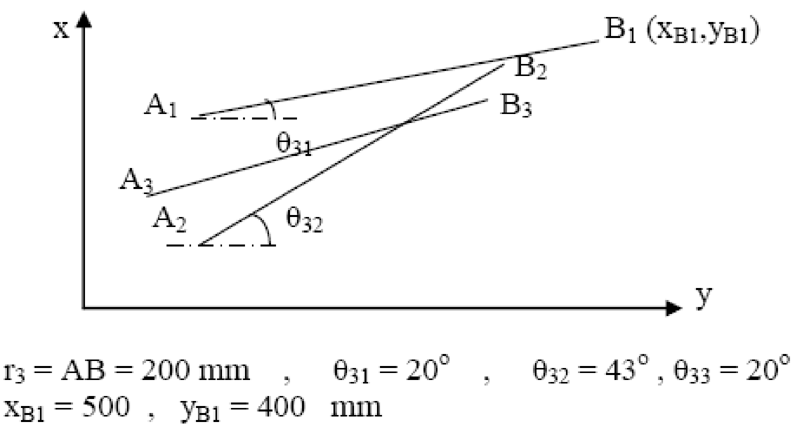

Fig.3: Desired 3-coupler positions.

Mechanism Synthesis:

A MATLAB code is written to solve Eqs.7-14 satisfying the right hand side which is zero for the 8 equations.

- Code input (guessed values of the unknown parameters): $r_{3n} = 5$, $r_{1n} = 6$, $\theta_{31} = 20^o$, $\theta_{32} = 43^o$, $\theta_{33} = 30^o$, $\mu_1 = \mu_2 = 90^o$
- Code output:

3.4544	(r_{4n})
3.0748	(θ_1)
1.5824	(θ_{21})
5.1043	(θ_{41})
4.9180	(θ_{22})
5.3648	(θ_{42})
3.6944	(θ_{23})
5.5251	(θ_{43})

- Values of the nonlinear functions:

0.0197 -0.0818 -0.0283 0.0868 0.0009 0.0005 -0.0429 0.0981

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- Mechanism dimensions:

Coupler length:	r_3 = 200 mm (required)
Crank length:	$r_2 = r_3/r_{3n}$ = 200/5 = 40 mm
Rocker length:	$r_4 = r_{4n} x r_2$ = 138.2 mm
Ground length:	$r_1 = r_{1n} x r_2$ = 240 mm
Ground angle:	$\theta_1 = 176.2^o$
Crank orientation:	$\theta_{21} = 90.66^o$
	$\theta_{22} = 281.8^o$
	$\theta_{23} = 211.7^o$
Rocker orientation:	$\theta_{41} = 292.4^o$
	$\theta_{42} = 307.4^o$
	$\theta_{43} = 316.5^o$

- The designed mechanism in its three positions is shown in Fig.4.

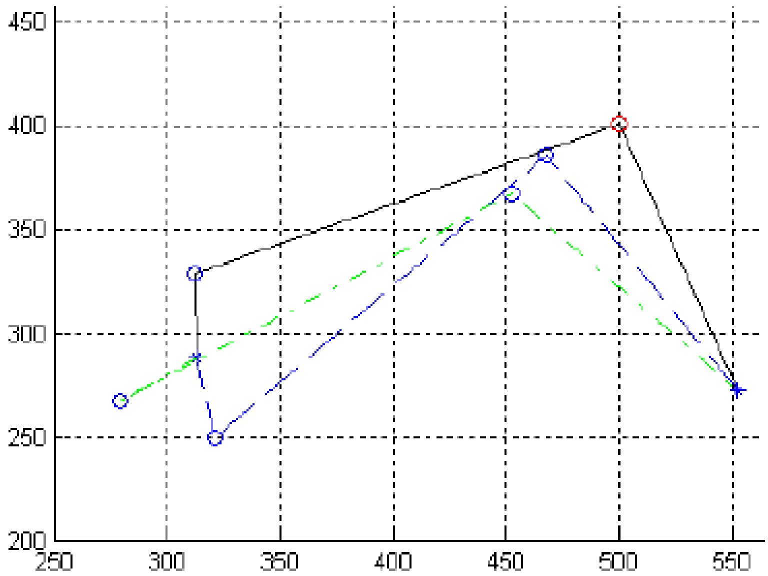

Fig.4: The synthesized 4-bar mechanism in its 3 positions.

- Coordinates x_{B1} and y_{B1} are used to locate the fixed pivot Q in the xy-plane (Fig.2).
- Transmission angles (μ) of the designed mechanism:

In the first position:	$\mu_1 = 92.4^o$
In the second position:	$\mu_2 = 84.4^o$
In the third position:	$\mu_3 = 106.5^o$

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- Mechanism type:

L_{min} = 40 mm (crank)
L_{max} = 240 mm
L_a = 138.2 mm
L_b = 200 mm
$L_{min} + L_{max}$ = 280 mm
$L_a + L_b$ = 338.2 mm

Then: $L_{min} + L_{max} < L_a + L_b$ and the crank is the minimum.

Therefore, the designed mechanism is a crank-rocker Grashof mechanism [17].

3. CONCLUSIONS

* The proposed approach is very accurate and reliable in synthesizing 4-bar planar mechanisms for 3 specific positions of its coupler.
* The assumptions are only one dimension (r_1) giving easy and straight forward design of the 4-bar mechanism.
* The coupler traces exactly the desired 3-positions.
* The deviation of the transmission angle of the mechanism from the ideal value of 90^o is:
  - 2.7 % error in the first coupler position.
  - 6.2 % error in the second coupler position.
  - 18.4 % error in the third coupler position.
* The large deviation in the third mechanism position is because its transmission angle was not included in the model equations. However, its value is within the recommended range of $45^o \le \mu \le 135^o$ [18].

4. NOMENCLATURES

$f_1, f_2, \ldots, f_8$: nonlinear mechanism functions.
r_1, r_2, r_3, r_4: lengths of links 1, 2, 3 and 4.
r_{1n}, r_{3n}, r_{4n}: normalized lengths of links 1, 3 and 4.
x_{A1}, y_{A1}: coordinates of point A_1.
x_{A2}, y_{A2}: coordinates of point A_2.
x_{A3}, y_{A3}: coordinates of point A_3.
$x_1, x_2, \ldots, x_8$: mechanism unknown parameters.
μ_1: mechanism transmission angle in the first coupler position.
μ_2: mechanism transmission angle in the second coupler position.
μ_3: mechanism transmission angle in the third coupler position.
θ_1: orientation of link 1 (frame).
$\theta_{21}, \theta_{31}, \theta_{41}$: orientation of links 2, 3 and 4 in the first mechanism position.
$\theta_{22}, \theta_{32}, \theta_{42}$: orientation of links 2, 3 and 4 in the second mechanism position.
$\theta_{23}, \theta_{33}, \theta_{43}$: orientation of links 2, 3 and 4 in the third mechanism position.

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