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SIMILARITY MEASURE BETWEEN NEUTROSOPHIC REFINED SETS AND THEIR APPLICATIONS IN MEDICAL DIAGNOSIS

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ABSTRACT
Neutrosopohic refined set is an important extension of neutrosophic set. In this paper, we focus on introducing similarity measure between neutrosophic refined sets based on the exponential operation. The proposed similarity measure provides a new way to handle the indeterminate and inconsistent information. Also we have examined some relevant properties of similarity measure between neutrosophic refined sets based on exponential operation. Finally, an application of neutrosophic refined set is given in medical diagnosis problems to illustrate the benefit of the proposed approach.

Keywords: Neutrosophic set, Neutrosophic refined set, Exponential similarity measure, medical diagnosis.

1. INTRODUCTION
Many of the real life problems like medical sciences, economics, engineering etc., involve imprecise data and their decisions involve the use of mathematical principles based on ambiguity and uncertainty.. Neutrosophic set (NS) is a generalized set of fuzzy set [8] and intuitionistic fuzzy set [1]. The neutrosophic set, introduced by F.Smarandache [8], has a powerful tool to deal with uncertain, imprecise, incomplete and inconsistent information. In 2005, Wang et al [14] introduced the instance of neutrosophic sets known as single valued neutrosophic sets (SVNS) that can be used in a real engineering application from the practical point of view. Although, in many applications, the decision information may be provide as sequence, due to lack of knowledge or data about the problem domains. Thus Deli and Broumi [4] introduced the concept of neutrosophic refined sets and studied some of their basic properties. Thus, neutrosophic refined sets, as a useful generation of NS, which is characterized by a membership sequence, indeterminate-membership sequence and falsity-membership sequence, whose values are sequence rather than real numbers. Also neutrosophic refined set can represent uncertain, imprecise, incomplete and inconsistent information which exist in the real world. As an important extension of NS, neutrosophic refined set have many applications in real life.
Recently, several theories have been proposed to deal with uncertainty, imprecision and vagueness. In some situations, there is the possibility of each element having different truth, indeterminate and false membership functions. The distinctive characteristic of neutrosophic set is that it contains multi truth, indeterminate and false membership. A similarity measure for neutrosophic set is used for estimating the degree of similarity between two neutrosophic sets. Many methods have been proposed for measuring the degree of similarity between neutrosophic set, S.Broumi and F.Smarandache [2] proposed several definitions of similarity between NS.
Several similarity measures are discussed in literature [2-6]. Smarandache [9] proposed several definitions of similarity measure between neutrosophic sets. P.Majumdar and S.K.Samanta [6] recommended some new methods for measuring the similarity between neutrosophic set. Recently, Deli and Broumi [4] introduced the concept of neutrosophic refined sets and studied some of their basic

properties. The concept of neutrosophic refined sets (NRS) is a generalization of fuzzy multisets and intuitionistic fuzzy multisets. Broumi and Smarandache [2] proposed the cosine similarity measure of neutrosophic refined sets. Mondal and Pramanik [5] proposed the cotangent similarity measure of neutrosophic refined sets.

The existing similarity measures adopt different concepts with its own strength and weakness. But in most demanding situations, most of the existing similarity measures fail to calculate the similarity measure correctly. To overcome the drawbacks of existing similarity measures, this work aims to propose a similarity measure between neutrosophic refined set based on exponential operation and apply them to medical diagnosis problems, in order to validate the supportive evidence. The article was structured as follows. Section 2, presents the basic definitions and notion of neutrosophic set and neutrosophic refined set. Section 3 deals with proposed definitions and its relevant properties. Further in Section 4 we present an application of similarity measure between neutrosophic refined sets in medical diagnosis problems. Finally the conclusion was given in section 5.

2. PRELIMINARIES

This section gives a brief overview of the concepts of neutrosophic set, and neutrosophic refined sets

2.1 Neutrosophic Sets

Definition 1[14]

In a neutrosophic set A^N in a universal set X, its characteristic functions are expressed by a truth-membership function $T_A^N(x)$, an indeterminacy-membership function $I_A^N(x)$, and a falsity-membership function $F_A^N(x)$, respectively. The functions $T_A^N(x)$, $I_A^N(x)$, $F_A^N(x)$ in X are real standard or nonstandard subsets of$]^-0, 1^+[$, i.e., $T_A^N(x): X \to]^-0, 1^+[$, $I_A^N(x): X \to]^-0, 1^+[$ and $F_A^N(x): X \to]^-0, 1^+[$. Then, the sum of $T_A^N(x)$, $I_A^N(x)$ and $F_A^N(x)$ is no restriction, i.e., $^-0 \le \sup T_A^N(x) + \sup I_A^N(x) + \sup F_A^N(x) \le 3^+$.

2.2 Neutrosophic refined set

A. Definition 2[4]

Let X be a universe, a neutrosophic refined set on X can be defined as follows:

$$A = \left\{ (x, (T_A^1(x), T_A^2(x), ..., T_A^p(x)), (I_A^1(x), I_A^2(x), ..., I_A^p(x)), (F_A^1(x), F_A^2(x), ..., F_A^p(x))) : x \in X \right\}$$

where $T_A^1(x), T_A^2(x), ..., T_A^p(x) : X \to [0,1]$, $I_A^1(x), I_A^2(x), ..., I_A^p(x) : X \to [0,1]$ and $F_A^1(x), F_A^2(x), ..., F_A^p(x) : X \to [0,1]$ such that $0 \le T_A^j(x) + I_A^j(x) + F_A^j(x) \le 3$ for j=1, 2, …, p for any $x \in X$, $(T_A^1(x), T_A^2(x), ..., T_A^p(x)), (I_A^1(x), I_A^2(x), ..., I_A^p(x))$ & $(F_A^1(x),, F_A^2(x), ..., F_A^p(x))$ is the truth-membership sequence, indeterminate-membership sequence and falsity-membership sequence of the element x, respectively. Also, p is called the dimension of neutrosophic refined set (NRS) A.

B. Definition 3 [4]

Let $A = \left\{ (x, (T_A^1(x), T_A^2(x), ..., T_A^p(x)), (I_A^1(x), I_A^2(x), ..., I_A^p(x)), (F_A^1(x),, F_A^2(x), ..., F_A^p(x))) : x \in X \right\}$ and $B = \left\{ (x, (T_B^1(x), T_B^2(x), ..., T_B^p(x)), (I_B^1(x), I_B^2(x), ..., I_B^p(x)), (F_B^1(x), F_B^2(x), ..., F_B^p(x))) : x \in X \right\}$ be two neutrosophic refined set then $A \subseteq B \Rightarrow T_A^j(x) \le T_B^j(x), I_A^j(x) \ge I_B^j(x) \,\&\, F_A^j(x) \ge F_B^j(x)$ for all $x \in X$.

2.3 Similarity measure

Definition 4 [4]

Let A and B be two neutrosophic refined sets, then

1. $S(A,B) \in [0,1]$
2. $S(A,B) = S(B,A)$
3. $S(A,B) = 1$ if and only if $A=B$

4. If $A \subseteq B \subseteq C$, then $S(A,C) \leq S(A,B)$ and $S(A,C) \leq S(B,C)$

3. PROPOSED SIMILARITY MEASURE BETWEEN NEUTROSOPHIC REFINED SETS BASED ON EXPONENTIAL OPERATION

Let $A = \left\{ (x,(T_A^1(x),T_A^2(x),...,T_A^p(x)),(I_A^1(x),I_A^2(x),...,I_A^p(x)),(F_A^1(x),,F_A^2(x),...,F_A^p(x))) : x \in X \right\}$ and $B = \left\{ (x,(T_B^1(x),T_B^2(x),...,T_B^p(x)),(I_B^1(x),I_B^2(x),...,I_B^p(x)),(F_B^1(x),F_B^2(x),...,F_B^p(x))) : x \in X \right\}$ be two neutrosophic refined set then the exponential similarity measure is defined as

$$S_R(A,B) = 1 - \frac{1}{n}\sum_{i=1}^{n}\left[\alpha(x_i)e^{1-\alpha(x_i)}\right] \qquad \text{--------------------(1)}$$

where $\alpha(x_i) = \frac{1}{3p}\sum_{j=1}^{p}\left[\left|T_A^j(x_i) - T_B^j(x_i)\right| + \left|I_A^j(x_i) - I_B^j(x_i)\right| + \left|F_A^j(x_i) - F_B^j(x_i)\right|\right]$

where p is called the dimension of neutrosophic refined set.

3.1 Properties

(i) $S_R(A,B) > 0$

Proof:

As the membership, indeterminate and non-membership functions of the neutrosophic refined set lies between 0 and 1, hence $S_R(A,B)$ also lies between 0 and 1.

(ii) $S_R(A,B) = S_R(B,A)$

Proof:

It is observed that,

$$\left|T_A^j(x_i) - T_B^j(x_i)\right| = \left|T_B^j(x_i) - T_A^j(x_i)\right|$$

$$\left|I_A^j(x_i) - I_B^j(x_i)\right| = \left|I_B^j(x_i) - I_A^j(x_i)\right|$$

$$\left|F_A^j(x_i) - F_B^j(x_i)\right| = \left|F_B^j(x_i) - F_A^j(x_i)\right|$$

$$S_R(A,B) = 1 - \frac{1}{n}\sum_{i=1}^{n}\left[\frac{1}{3p}\sum_{j=1}^{p}\left[\left|T_A^j(x_i) - T_B^j(x_i)\right| + \left|I_A^j(x_i) - I_B^j(x_i)\right| + \left|F_A^j(x_i) - F_B^j(x_i)\right|\right] e^{1-\frac{1}{3p}\sum_{j=1}^{p}\left[\left|T_A^j(x_i) - T_B^j(x_i)\right| + \left|I_A^j(x_i) - I_B^j(x_i)\right| + \left|F_A^j(x_i) - F_B^j(x_i)\right|\right]}\right]$$

$$= 1 - \frac{1}{n}\sum_{i=1}^{n}\left[\frac{1}{3p}\sum_{j=1}^{p}\left[\left|T_B^j(x_i) - T_A^j(x_i)\right| + \left|I_B^j(x_i) - I_A^j(x_i)\right| + \left|F_B^j(x_i) - F_A^j(x_i)\right|\right] e^{1-\frac{1}{3p}\sum_{j=1}^{p}\left[\left|T_B^j(x_i) - T_A^j(x_i)\right| + \left|I_B^j(x_i) - I_A^j(x_i)\right| + \left|F_B^j(x_i) - F_A^j(x_i)\right|\right]}\right]$$

$$= S_R(B,A)$$

(iii) If $A \subseteq B \subseteq C$, then $S_R(A,C) \le S_R(A,B)$ and $S_R(A,C) \le S_R(B,C)$

Proof:

By definition 3, since $A \subseteq B \subseteq C$

$T_A^j(x) \le T_B^j(x) \le T_C^j(x)$

$I_A^j(x) \ge I_B^j(x) \ge I_C^j(x)$

$F_A^j(x) \ge F_B^j(x) \ge F_C^j(x)$

Hence ,

$$\left|T_A^j(x_i) - T_B^j(x_i)\right| \le \left|T_A^j(x_i) - T_C^j(x_i)\right|$$

$$\left|T_B^j(x_i) - T_C^j(x_i)\right| \le \left|T_A^j(x_i) - T_C^j(x_i)\right|$$

$$\left|I_A^j(x_i) - I_B^j(x_i)\right| \le \left|I_A^j(x_i) - I_C^j(x_i)\right|$$

$$\left|I_B^j(x_i) - I_C^j(x_i)\right| \le \left|I_A^j(x_i) - I_C^j(x_i)\right|$$

$$\left|F_A^j(x_i) - F_B^j(x_i)\right| \le \left|F_A^j(x_i) - F_C^j(x_i)\right|$$

$$\left|F_B^j(x_i) - F_C^j(x_i)\right| \le \left|F_A^j(x_i) - F_C^j(x_i)\right|$$

Hence, the proposed exponential similarity measure is a decreasing function.

Therefore, $S_R(A,C) \le S_R(A,B)$ and $S_R(A,C) \le S_R(B,C)$

4. APPLICATION TO MEDICAL DIAGNOSIS PROBLEM [3]

This section presents an application of neutrosophic refined set in medical diagnosis. Let us consider S is a set of symptoms, D is a set of diseases and P is a set of patients. Let Q be a neutrosophic refined relation from the set of patients to the symptoms. i.e., $Q(P \to S)$ and R is a neutrosophic relation from the set of symptoms to the disease i.e., $R(S \to D)$.

Let the four patients $P = \{P_1, P_2, P_2, P_4\}$ and the set of symptoms S={Temperature, Headache, Stomach pain, Cough, Chest pain}. The Neutrosophic Refined Relation $Q(P \to S)$ is given as in Table 1. Let the set of disease D={Viral fever, Malaria, Typhoid, Stomach problem, Chest problem}. The Neutrosophic Relation $R(S \to D)$ is given in table 2. The relation of patients and diseases is establishing using equation (1) is noted in table 3. Finally, maximum value of each row is the higher similarity and were selected to find the possibility of the patient P_k (k=1,2,3,4) was suffering from the disease D_r(r=1,2,3,4,5).

Table 1: Patient - symptom relation

Q	Temperature	Headache	Stomach pain	Cough	Chest pain
P_1	(0.8,0.1,0.1) (0.6,0.3,0.3) (0.6,0.3,0.1)	(0.6,0.1,0.3) (0.5,0.2,0.4) (0.5,0.1,0.2)	(0.2,0.8,0) (0.3,0.5,0.2) (0.2,0.3,0.4)	(0.6,0.1,0.3) (0.4,0.4,0.4) (0.4,0.3,0.3)	(0.1,0.6,0.3) (0.3,0.4,0.5) (0.2,0.5,0.4)
P_2	(0,0.8,0.2) (0.2,0.6,0.4) (0.1,0.6,0.4)	(0.4,0.4,0.2) (0.5,0.4,0.1) (0.4,0.6,0.3)	(0.6,0.1,0.3) (0.4,0.2,0.5) (0.3,0.2,0.4)	(0.1,0.7,0.2) (0.2,0.7,0.5) (0.3,0.5,0.4)	(0.1,0.8,0.1) (0.3,0.6,0.4) (0.3,0.6,0.4)
P_3	(0.8,0.1,0.1) (0.6,0.4,0.1) (0.5,0.3,0.3)	(0.8,0.1,0.1) (0.6,0.2,0.4) (0.6,0.1,0.3)	(0,0.6,0.4) (0.2,0.5,0.5) (0.3,0.4,0.6)	(0.2,0.7,0.1) (0.2,0.5,0.5) (0.1,0.6,0.3)	(0,0.5,0.5) (0.2,0.5,0.3) (0.3,0.3,0.4)
P_4	(0.6,0.1,0.3) (0.4,0.3,0.2) (0.5,0.2,0.3)	(0.5,0.4,0.1) (0.4,0.4,0.4) (0.5,0.2,0.4)	(0.3,0.4,0.3) (0.2,0.4,0.5) (0.1,0.5,0.4)	(0.7,0.2,0.1) (0.5,0.2,0.4) (0.6,0.4,0.1)	(0.3,0.4,0.3) (0.4,0.3,0.4) (0.3,0.5,0.5)

Table 2: Symptom – disease relation

R	Viral fever	Malaria	Typhoid	Stomach problem	Chest problems
Temperature	(0.6,0.3,0.3)	(0.2,0.5,0.3)	(0.2,0.6,0.4)	(0.1,0.6,0.6)	(0.1,0.6,0.4)
Headache	(0.4,0.5,0.3)	(0.2,0.6,0.4)	(0.1,0.5,0.4)	(0.2,0.4,0.6)	(0.1,0.6,0.4)
Stomach pain	(0.1,0.6,0.3)	(0,0.6,0.4)	(0.2,0.5,0.5)	(0.8,0.2,0.2)	(0.1,0.7,0.1)
Cough	(0.4,0.4,0.4)	(0.4,0.1,0.5)	(0.2,0.5,0.5)	(0.1,0.7,0.4)	(0.4,0.5,0.4)
Chest pain	(0.1,0.7,0.4)	(0.1,0.6,0.3)	(0.1,0.6,0.4)	(0.1,0.7,0.4)	(0.8,0.2,0.2)

Table 3: Diagnosis based on the neutrosophic exponential similarity measure.

T	Viral fever	Malaria	Typhoid	Stomach problem	Chest problems
P_1	0.8150	0.8001	0.8016	**0.8188**	0.8117
P_2	0.8000	0.8001	**0.8093**	0.8064	0.8043
P_3	0.8023	0.8032	0.8000	0.8161	**0.8194**
P_4	0.8110	0.8001	0.8002	**0.8187**	0.8064

From Table 3, it is observed that, if the doctor agrees, then P_1, P_4 suffers from Stomach problem, P_2 suffers from Typhoid and P_3 suffer from Chest problem.

5. CONCLUSION

This paper defines a new exponential similarity measure between neutrosophic refined sets and investigates some of their basic properties. We have presented an application of exponential similarity measures of neutrosophic refined sets in medical diagnosis. The proposed method can be applied in the areas such as clustering, image processing etc., In the future; we will enhance this method to other type of neutrosophic sets.

REFERENCES

[1] Atanassov, K 1986, ‘ Intuitionistic fuzzy sets’, Fuzzy Sets and Systems, Vol. 20, no. 1, pp. 87–96.

[2] S.Broumi and F.Smarandache, Neutrosophic refined similarity measure based on cosine function, Neutro Sets and Syst., 6 (2014) 42-48.

[3] S.Broumi and F.Smarandache, Extended Hausdorff distance and similarity measures for neutrosophic refined sets and their applications in medical diagnosis, Journal of New Theory, 7 (2015) 64-78.

[4] I.Deli, S.Broumi and F.Smarandache, On neutrosophic refined sets & their applications in medical diagnosis, Journal of New Theory, 6 (2015) 88-98.

[5] K.Mondal and S.Pramanik, Neutrosophic refined similarity measure based on cotangent function and its application to multi-attribute decision making, Global Journal of Adv Res, 2(2) (2015) 486-496.

[6] K.Mondal and S.Pramanik, Neutrosophic refined similarity measure based on tangent function and its application to multi-attribute decision making, Journal of NewTheory, 8 (2015) 41-50.

[7] P. Majumdar, S.K. Samant,” On similarity and entropy of neutrosophic sets”, Journal of Intelligent and Fuzzy Systems, 1064-1246(Print)-1875-8967(Online), (2013), DOI:10.3233/IFS-130810, IOS Press.

[8] Z.Pawlak, Rough sets, Int. Journal of Infor. and Comp. Sci., 11 (1982) 341-356.

[9] F.Smarandache, A unifying field in logics, Neutrosophy: Neutrosophic probability, set and logic, Rehoboth: Amer Res Press (1998).

[10] F.Smarandache, n-valued refined neutrosophic logic & its application in Physics, Prog. in Phy, 4 (2013)143-146.

[11] S.Sebastian and T.V.Ramakrishnan, Multi fuzzy sets, Int. Math Forum, 5(50) (2010) 2471-2476.

[12] S.Sebastian and T.V.Ramakrishnan, Multi fuzzy sets: an extension of fuzzy sets, Fuzzy Infor Engi, 3(1) (2011) 35-43.

[13] T.K.Shinoj and S.J.John, Intuitionistic fuzzy multisets and its application in medical diagnosis, Wor Aca of Sci, Engi and Tech, 61 (2012) 1178-1181.

[14] Wang, H, Smarandache, F, Zhang, Y Q & Sunderraman, R 2005, Interval neutrosophic sets and logic; theory and applications in computing, Phoenix, Arizona.

[15] R.R.Yager, On the theory of bags (Multi sets), Int. Journal of General System, 13 (1986) 23-37.

[16] J.Ye, Vector similarity measures of simplified neutrosophic sets and their application in multicriteria decision making, Int. Journal of Fuzzy Syst, 16(2) (2014) 204-215.

[17] L.A.Zadeh, Fuzzy sets, Infor. and Cont., 8 (1965) 338-353.